PRINCIPLES OF FINANCIAL ENGINEERING

Answers to Exercises

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This first draft will be revised and the answers will be extended as comments are received.
Chapter 1

Introduction

Case Study: Japanese Loans and Forwards

1. Follow Figures 1-1 and 1-2 from the text.

2. Japanese banks borrow in yen, and buy spot dollars from their Western counterparties. So, the Western banks are left holding the yen for the time of the loan (three months, in this case).

   The main point is here. In an FX transaction, in this case buying Yen, the purchased currency may have to be kept overnight in a Yen denominated account. The FX is by definition not EuroYen, so these accounts have to be in a bank Japan. some of these will be Japanese banks.

3. A nostro account is one that a bank holds with a “foreign bank”. (In this case London banks hold Nostro accounts with Japanese Banks in Tokyo, for example.) Nostro accounts are usually in the currency of the foreign country. Suppose an American bank called Bank A buys Euros from an European bank Bank B. These Euros cannot “leave” Europe. They will be sent to a European bank, say Bank of Europe, to be kept in a Deposit account for the use of Bank A. This would be a nostro account of Bank A. Bank A will have similar nostro accounts in Japan, Australia, etc... to trade Dollar against Yen or Australian dollar.

   This allows for easy cash management because the currency doesn’t need to be converted. Incidentally, nostro is derived from the Latin term “ours”.

   The Western banks may not be willing to hold the Yen in their nostro accounts because this requires them to hold capital against the yen for regulatory purposes.

   Japanese banks being more risky, risk managers may also be against holding “too much” in a Nostro account in Japan. Note that banks operate in an environment where others have credit lines against each other. The “Headquarters” may not want a currency desk to have
exposure to Japanese Banks beyond a certain limit. This may force Western banks to dump the excess Yens at a negative interest rate.

4. By not holding the yen, the Western banks could potentially lose significant sums if the bank where the Nostro account is held defaults. For this reason they may prefer to dump the yen deposits and earn negative yield because they can be more than compensated with their earnings from the spot-forward trade.
Question 1

Going by swap market conventions, the fixed payments for fixed payer swaps are:

- $100 \times .0506 \times 1 = \text{USD } 5.06 \text{ million per year}$
- $100 \times .0506 \times 1 = \text{Euro } 5.06 \text{ million per year}$

Fixed payments for the fixed receiver swaps are:

- $100 \times .0510 \times 0.5 = \text{JPY } 2.55 \text{ million per 6 months}$
- $100 \times .0510 \times 0.5 = \text{GBP } 2.55 \text{ million per 6 months}$

Question 2

(a) One could take a market arbitrage position as follows: buy Honeywell shares and sell General Electric shares. If the merger takes place, the Honeywell shares will convert to GE shares - that is, these shares will become similar and now one can sell the expensive shares and make a profit.

(b) You do not need to deposit funds to take this position.

(c) You could borrow funds for this position. You would need to if you do not have any GE shares. If you had them then you could engineer this short position through short selling them.

(d) This is different from the academic sense of the word arbitrage. That involves zero risk and infinite gain. Here we do face a risk (see below) and our gains might be very high - but not infinite!

(e) You would be taking the risk that the merger indeed goes through successfully.
Question 3

(a) The dealers are selling the Matif contract and buying the Liffe. (See below)

(b) The horizontal axis would have price and the vertical axis would show gain and loss.

(c) Since both Euribor and Euro BBA Libor are both European based rates, the profit would simply be scaled down - if all European interest rates would be dramatically lowered.
Figure 1:
Chapter 3

CASH FLOW ENGINEERING AND FORWARD CONTRACTS

EXERCISES

Question 1

(a) Before FAS 133, if companies qualified for hedge accounting, their hedges were assumed to be perfect—no valuation or testing required. Now, under FAS 133, risk managers seeking hedge accounting treatment have to thoroughly document each hedge from the outset and explain why they are undertaking the transaction. They have to mark their derivatives to market every quarter (no small feat for many instruments), then prove they are effectively hedging the underlying exposure.

It’s this sense of having to pay for the sins of others that accounts for the deep resentment toward FAS 133. Many finance executives suspect the new rules have less to do with improving financial statements than with discouraging treasury departments from speculating with derivatives.

(b) Constructing synthetic swaps will involve replication of a swap by portfolios of bonds. These do not come under the considerations of FAS 133. So all the work that FAS 133 brings with it needs not be done now.

Question 2

(Parts (a)-(c) answered together:) A gold miner risks losing money if the price of gold declines, between the time say, when she is mining the gold and when she would actually sell it. So, she sells futures. If the market prices fall, she has still locked in a rate (at the present time, based on the present day value of gold) high enough for her to make some profit on. This is how she can hedge against a steady decline in gold prices over the years.

Unless she sets a futures price that is lower than the present day value of gold, she cannot have a loss. And this will typically not happen since this would also lead to arbitrage opportunities. But the hedge could lead to a ‘loss’ in the sense that if the market price appreciates then she would not make as much profit as she could have.
Question 3

(a) Synthetic for Contract A involves:

‘Sell EUR (to get USD)’ is equivalent to

- **Loan**: Borrow EUR at \( t_0 \) for maturity \( t_1 \)
- **Spot operation**: Buy USD against EUR
- **Deposit**: Deposit USD at \( t_0 \) for maturity at \( t_1 \)

Here, \( t_0 \) is March 1, 2004 and \( t_1 \) is March 15, 2004. The underlying sum sold is 1,000,000 EUR.

Synthetic for Contract B involves:

‘Buy EUR against USD’ is equivalent to

- **Loan**: Borrow USD at \( t_0 \) for maturity \( t_1 \)
- **Spot operation**: Buy EUR against USD
- **Deposit**: Deposit EUR at \( t_0 \) for maturity at \( t_1 \)

Here, \( t_0 \) is March 1, 2004 and \( t_1 \) is April 30, 2004. The underlying sum bought is 1,000,000 EUR.

(b) In this part of the question, if we have correctly identified our synthetics we can simply interpret the data given to us in the question and use the pricing equation (8) from Section 5 of this Chapter. This is given by

\[
F_t = e^{t_0} \frac{B(t_0, t_1)^{eur}}{B(t_0, t_1)^{usd}}
\]

Consider Contract A and its synthetic from the previous part of this question above. We borrow EUR at 2.36%, buy USD at spot rate 1.1500 and deposit USD at 2.25%. So,

\[
F_t^1 = 1.1500 \times \frac{2.36}{2.25} = 1.2062
\]

Now, consider Contract B and its synthetic from the previous part of this question above. We borrow USD at 2.27%, buy EUR at spot rate
1.1505 and deposit EUR at 2.35%. So,

\[ F_t^2 = 1.1505 \times \frac{2.35}{2.27} = 1.1910 \]

(c) The basic idea is as follows: now the outright forward spot rate is 1.1510/1.1525. With this new rate, consider both synthetics. Long the one that gives you higher profit and short the other. This will give arbitrage.

**Question 4**

To rank the instruments we need to recall the conventions from Chapter 2. We review Section 5 from Chapter 2, and Table 2-1 in particular. According to the formula given there, we first calculate present day values of these instruments.

- **30-day US T-bill:** Day count convention: ACT/360. Yield is quoted at discount rate, so we have

  \[ B(t, T) = 100 - R_T \left( \frac{T - t}{365} \right) 100 = 100 - 5.5 \left( \frac{30}{365} \right) = 99.5479 \]

- **30-day UK T-bill:** Day count convention: ACT/365. Yield is quoted at discount rate, so we have

  \[ B(t, T) = 100 - R_T \left( \frac{T - t}{365} \right) 100 = 100 - 5.4 \left( \frac{30}{365} \right) = 99.5561 \]

- **30-day ECP:** Day count convention: ACT/360. Yield is quoted at the money market yield, so we have

  \[ B(t, T) = \frac{100}{1 + R_T \left( \frac{T - t}{365} \right)} = \frac{100}{1 + 0.052 \left( \frac{30}{365} \right)} = 99.5744 \]

- **30-day interbank deposit USD:** Day count convention: ACT/360. Yield is quoted at the money market yield, so we have

  \[ B(t, T) = \frac{100}{1 + R_T \left( \frac{T - t}{365} \right)} = \frac{100}{1 + 0.055 \left( \frac{30}{365} \right)} = 99.5500 \]
• **30-day US CP**: Day count convention: ACT/360. Yield is quoted at the discount rate, so we have

\[
B(t, T) = 100 - R^T \left( \frac{T - t}{365} \right) 100 = 100 - 5.6 \left( \frac{30}{365} \right) = 99.5397
\]

Yields on these instruments = 100 − B(t, T), so to arrange these instruments in increasing order of their yields, we simply arrange them in decreasing order of their present day values.

(a) Since we are dealing with an ECP (Euro), the day count convention used is ACT/360. So there are 62 days till maturity. Also, we have to use the money market yield rate to compute the present day value. (We have again used conventions from Chapter 2, Table 2-1). So,

\[
B(t, T) = 100 - R^T \left( \frac{T - t}{365} \right) 100 = 100 - 3.2 \left( \frac{62}{365} \right) = 99.4564384
\]

is the present day of a bond that would yield 100 USD. So, we have to make a payment of

\[
99.4564384 \times \frac{10,000,000}{100} = 9945643.84
\]

US Dollars for this ECP.
Chapter 3

Cash Flow Engineering and Forward Contracts

Case Study: HKMA and Hedge Funds, Summer of 1998

The answer below is based on a report submitted by students in the Financial Engineering Class at ISMA Centre. The text quoted from these answers are in italics.

General Background

Hong Kong is a special administrative region of China (HKSAR). It is, arguably, the world’s fourth-largest center for international finance after New York, London and Tokyo.

Hong Kong is the world’s ninth-largest international banking center and the second largest in Asia, in term of the value of external transactions.

Hong Kong has the world’s tenth-largest securities market and the second-largest in Asia after Tokyo.

Hong Kong’s gross domestic product (GDP) grew by 3% in 1999—compared with a decline of 5.1% in 1998 (the crisis year)—and grew by a further 14.3% in the first quarter of 2000.

There is no central bank in Hong Kong, instead, the monetary system is one of Currency Board. It is called the Hong Kong Monetary Authority (HKMA). Traders refer to it as “the M-A”. HKMA was created in 1993 through a merger of the office of the Exchange Fund and the office of the Commissioner of Banking.

The primary monetary policy of Hong Kong Monetary Authority (HKMA) is to maintain exchange rate stability within the framework of Linked Exchange Rate System through sound management of the exchange fund, monetary operations and other means deemed necessary.
The Financial Sector in Hong Kong

The financial sector is a key element in the future growth of Hong Kong and a key element in Hong Kong's financial sector is the exchange regime adopted by the Hong Kong government. Hong Kong currency is pegged to the US dollar. This is simply referred to as “The peg”.

HKMA doesn't issue bank notes, bank notes are issued by the three note-issuing commercial banks (NIBs). When the three NIBs issue bank notes, they are required to submit US dollar (at HK$ 7.8 = US$ 1) to the HKMA for the account of the exchange fund in return for non-interest bearing certificates for indebtedness (which are required by law as backing for the bank notes issued). The Hong Kong dollar banknotes are therefore fully backed by US dollar held by the Exchange Fund.

This requirement that notes and coins be fully backed by foreign exchange reserves means that the exchange rate system operates as an example of currency board.

Currency board: is a legal framework that enables local currency to be issued only under strictly limited circumstances. The goal is to ensure that local currency is at all time fully or almost fully backed by reserves of strong currency such as the U.S dollar, so that the two become nearly perfect substitutes. It involves a legislative commitment to exchange the two currencies at a fixed rate, combined with restrictions on the issuing authority such that new local currency is issued only in the presence of sufficient reserves of external currency, so that the exchange commitment is always credible.

In Hong Kong the system differs from pure currency board arrangement, for two reasons:

(1) The HKMA has a limited scope for affecting monetary conditions.

(2) Only transaction related with note-issuing purposes between the Exchange Fund and the Note-Issuing Banks (NIBs) are carried out at a linked exchange rate, whereas all other transactions are conducted freely negotiated rates.

The principal features of a currency board are as follows:
• Discretionary policy in setting the exchange rate is removed.

• Discretionary policy in determining the money supply is limited since increases in the supply of notes in circulation are tied to increases in bank holdings of US dollars.

• Interest rate policy is used for the purpose of augmenting the demand for local currency. For example, a fall in the demand for Hong Kong dollars can be partly offset by increases in the interest rates in order to encourage the holding of Hong Kong bank deposits rather than repatriating the currency for deposits in overseas banks.

Thus, although the currency board system provided stability to the Hong Kong dollar exchange rate, and also to the domestic money supply, it limits the use of monetary policy for other purposes.

In addition, the choice of the “link currency” will also have an impact on the domestic economy. Movements in the rate of exchange of that currency with other currencies will affect the exchange rate between the Hong Kong dollar and those other currencies.

What is a Hedge Fund?

A Hedge Fund is a fund established by one or several partners with net worth of at least $1 million (although this maybe falling). It uses long and short positions to take speculative positions in multiple markets simultaneously. (Regular equity funds are not allowed by law, to short securities.) Hedge funds use leverage and trade derivatives in order to maximize returns.

After the leverage effect Hedge Funds command large amounts of resources. Their positions can significantly affect markets, particularly those markets that are relatively less liquid.

Hedge Fund have been playing a no-lose game

In the simplest strategy a hedge fund borrows Hong Kong dollars(HKD) and then sells them in the market against USD, i.e., they short the HKD. Note that this will cause the money supply to shrink. A decrease in money supply leads to an interest rates increase. Increases in interest rates have several effects on the stock market. First borrowing HKD to buy stocks becomes
Introduction of the linked exchange rate system (Oct 1983)

World stock market crash (Oct 1987)

Gulf crisis (Aug 1990)

Closure of BCCI (HK) (Summer 1991)

ERM turmoil (Sep 1992)

Mexican crisis (Jan 1995)

Asian currency turmoil (July 1997–1998)

Figure 2:
more expensive. Hence fewer investors would use margin. Second, an increase in deposit interest rates will draw funds from stocks to deposits. Third, interest rate increases are negative for businesses and their value will go down. Again stocks decline.

On the other hand, higher interest rates lure more investors to park their money in Hong Kong, boosting the currency. But they also slam the stock market because rising rates hurt companies’ ability to borrow and expand.

However, many of these Hedge Funds involved in the speculation did not operate in the cash market. Instead they shorted the HKD in the futures markets. This does not require borrowing HKD. It is the counterparty who has to hedge the long HKD position who needs to “borrow HKD” from the banking system.

In the particular case discussed here Hedge Fund managers believed that they were taking little risk:

- The hedge funds bet on the collapse of the peg. If the peg breaks, the HKD is expected to fall. Given the psychology of those days, the casual view was that the HKD was overvalued. The only risk to Hedge Funds is that the peg holds.

  Under these conditions their loss will be the difference between the initial cost of entering the trade to sell HKD in futures markets and the pegged rate. The reading suggests that this cost is low.

**Example:** Hedge Fund enters contract to sell HK$ in six month’s. At expiration, the Hedge Fund needs to buy spot HKD and deliver these against the short future’s position.

  If the peg holds the cost of replacing the HKD it has sold is essentially the 6 month differential between USD and HKD interest rates.

  On Thursday August, 20th the difference in inter-bank interest rates was about 6.3%, (Hong Kong rates being higher due to heavy demand for HKD loans, which are needed to short the currency.) So a hedge fund manager making a USD 1 million bet Thursday against the HKD would have paid USD63,000.

  If the fund manager believed that the peg would break and thus the HKD depreciate, say, about 30%, then the potential profit would be USD300,000.
Compared to the cost of making the trade, USD63,000 this is a good profit.

**MA Intervenes**  HKMA intervened to defend the peg. Using its own FX reserves, MA sold USD. Normally, when a country with a pegged currency spends reserves to defend the currency’s value, the intervention will have to be “sterilized”. In other words, the central bank would buy local currency bonds from the banking system. The purchase will be roughly in similar quantities so that the overall monetary base remains constant.

However, doing this in Hong Kong at that time would result in further increases in interest rates. This would be considered as severely harmful by real estate companies in Hong Kong.

1. **PART A**

1.1 **What is the rationale of the double-play strategy?**

The hedge funds deploy a double-play strategy in order to engineer steep increases in interest rates and steep declines in stock prices so as to gain from their short positions in the stock market and in the FX futures market.

But first, some comments about the economic conditions prevailing at that time. In early August of 1998, external and domestic conditions deteriorated. The Dow Jones index declined sharply by 300 points on August 5th and the Yen was at an eight year low, at 147 on August 11th.

Rumors were abundant concerning abandonment of the peg. There was strong selling pressure on HKD early August.

1. Speculators shorted the HKD by swapping HKD for USD.
2. On the equity markets, the stocks index futures market open positions grew sharply:

   The HSI FUTURES rose from 70,000 contracts in June to 92,000 contracts in August.

The strategy of the Hedge Funds was to undermine the stability of the exchange value of the HK$ so as to produce sharply higher interest rates.
The sharp increases would then lower stock prices, it was hoped. Hedge Funds sell HKD. This increases HKD interest rates ($r$). Such high interest rates cannot be tolerated by property developers. Real Estate companies suffer serious losses and their stocks decline sharply. The HSI goes down, as the HIBOR goes up.

At this point, another strategy is to short sell borrowed shares. Yet, the existence of futures markets makes this redundant. A speculator can short the HSI index instead.

1.2 How are the HIBOR, HSI and HSI futures related?

The HIBOR and HSI are inversely related. Consequently futures on HIBOR and HSI are also inversely related. See Charts 2 and 3.
Chart 3
Differentials between Hong Kong dollar and US dollar interest rates
(July – September 1998)

Figure 4: Chart 3
1.3 Display the position explicitly

Example: I borrow 7,800,000 HKD at time $t = t_o$ at an interest rate $r_{t_o}$.

After one year I pay back $7,800,000(1 + r_{t_o})$.

At $t_o$ I exchange these HKD into USD1,000,000. These I deposit at the “risk free” rate $R_{t_o}$.

I expect that The HKD to depreciate 30% during the same time period.

My expected gain is:

$$R_{t_o}1,000,000 - r_{t_o} \frac{78,000}{e_{t_o}(1+.30)} + 300,000$$

where the $e_{t_o}$ is the pegged exchange rate, 7.8.

1.4 How is the position rolled over?

As Hong Kong government intervened Hedge Funds decide to just rolling their short position over during the month of August. These open positions are rolled by converting August contracts into September contracts.

The Table below gives further data for this case study.

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Chapter 4

ENGINEERING SIMPLE INTEREST RATE DERIVATIVES

EXERCISES

Question 1

(a) These are just the differences of the two prices. So, the mark to market losses are given by \(\{Q_1 - Q_0, Q_2 - Q_0, Q_3 - Q_0, Q_4 - Q_0, Q_5 - Q_0\}\). Of course, negative losses are gains.

(b) You just calculate the interest accrued after multiplying by \(1/360\) for every day and,

(c) Then adding the gains and losses.

Question 2

(a) Treasurer has risks for three months starting in three months. So a \(3 \times 6\) FRA is needed.

(b) To get the break even rate we need:

\[
\left(1 + .0673 \left(\frac{1}{4}\right)\right) \left(1 + f \left(\frac{1}{4}\right)\right) = \left(1 + .0787 \left(\frac{1}{2}\right)\right)
\]

(c) Lowest offered rate. (6.87%)  

(d) (FRA settlement) \((.0687 - .0609)(38\text{ million})(1/4)\)

Question 3

(a) The futures price has moved by 34 ticks. (It moved from \(Q_{t_0} = \$94.90\) to \(Q_{t_1} = \$94.56\).

(b) The current implied forward rate is given by

\[
\tilde{F}_{t_0} = \frac{100 - 94.90}{100} = 0.0510
\]
which means the buyer of the contract needs to deposit

\[ 100 \left( 1 - \frac{0.0510}{4} \right) = 98.725 \]
dollars per $100 dollars on expiry (which is in three months in this case)

(c) In three months the futures price moves to \( Q_{t_t} = 94.56 \) giving a implied forward rate of

\[ \tilde{F}_{t_t} = \frac{100 - 94.56}{100} = 0.0544 \]
and a settlement of

\[ 100 \left( 1 - \frac{0.0544}{4} \right) = 98.64 \]

So the buyer of the original contract receives a compensation as if she were making a deposit of $98.725 and receiving a loan of $98.64, making a loss of

\[ 98.64 - 98.725 = -0.085 \text{ per $100 dollars} \Rightarrow \text{Loss of $595000} \]
since the sum involved is $7 million.

**Question 4**

(a) The trader will buy (sell) the Libor-based FRA, and sell (buy) Tibor-based FRA. This way the market risk inherent in the Libor positions will be eliminated to a large degree. However, Tibor and Libor fixings occur at different times, so there still some risk in this position.

(b) Use two cash flow diagrams, one for Libor FRA the other for the Tibor FRA. In one case the trader is paying fixed and receiving floating. The other cash flow diagram will display the reserve situation. In this setting, the two fixed rates are known and their difference will remain fixed. The trader will have exposure to the difference between the floating rates.

(c) If Libor panel is made of better-rated banks, then the Libor fixings will be lower everything else being the same. This means that the spread between Libor and Tibor will widen. According to this, traders need to buy the spread if they decide to take such a speculative position.
Question 5

(a) To use the given data to create a $1 \times 4$ NZ $ FRA, the overall strategy would be:

- Replicate the forward borrowing in NZ $ by combining FX forwards at 1 month and 4 month with spot borrowing of A $ in the future (1 month - 4 month) plus a $1 \times 4$ A $ FRA.

- After we obtain the synthetic forward borrowing in NZ $ via the A $ FRA market, we retrieve the synthetic $1 \times 4$ NZ $ FRA.

The eventual complete contractual equation could be summarized as:

$1 \times 4$ NZ$ FRA equals:

1. Spot lending NZ $ at $t_1 = 1$ month till $t_2 = 4$ months at rate $L^{NZ}_{t_1}$.
2. Forward sale A $ at $t_1 = 1$ month.
3. Forward purchase of A $ at $t_2 = 4$ month.
4. Spot borrowing of A $ at $t_1 = 1$ month till $t_2 = 4$ month at rate $L^A_{t_1}$
5. $1 \times 4$ A $ FRA$

Note that we can leave out the spot lending and spot borrowing out of the contractual equation since they are spot operations.

(b) Left as exercise - use each of the 5 points from part (a) of the question to describe a cash flow.

(c) This position involves spot lending and borrowing in the future (1 to 4 month period) at the Libor rate. These spot operations bring with them additional credit and liquidity risks.

(d) Since a domestic FRA can be replicated by combining an FX FRA with forward currency transactions and spot lending and borrowing in the future, FRA markets and currency forwards should be related by some arbitrage relationships.

These arbitrage relationships are implicit in the contractual equation. The cost of locking in a future domestic borrowing cost should be equal
to the cost of combining the domestic and foreign positions that are required to build the synthetic FRA.

Question 6

(a) This can be done by taking a cash loan at time $t_0$, pay the Libor rate $L_{t_0}$, and buy a FRA strip made of two sequential FRA contracts - a $(3 \times 6)$ FRA and a $(6 \times 9)$ FRA. The cash flow diagrams are left as an exercise.

(b) Let $N$ be the sum to be borrowed. To find the fixed borrowing cost, simply add the costs incurred by:

- The $(3 \times 6)$ FRA, since $3.4 > 3.2$, so the floating rate is higher.
- The $(6 \times 9)$ FRA, since $3.7 > 3.2$, so the floating rate is higher.
- The cost from the three month fixed rate loan.

Additional Information on the Libor-Tibor Case Study

The following lists useful information concerning Libor, Tibor and Euribor. These information are obtained from Student answers to the case study, and from various official WEB sites. Two useful links are


Libor-based instruments play an important role in financial engineering. Readers are recommended to visit these WEB sites.

1. Japan Premium

The Japan premium reflects the fact that Japanese banking system is fragile due to balance sheets of Japanese banks. The domestic systematic risks leading to the Japan premium consist mainly in:

(a) solvency risk
(b) Liquidity risk

These are due to reduced profitability of Japanese banks and their retrenchment, to the closure of large troubled banks, to large undisclosed losses of banks during the 90’s.
2. **Tibor, Libor**

Let us introduce what is the BBA LIBOR-yen and the Euro-yen TIBOR. The BBA Yen LIBOR is the London Interbank Offer Rate fixed by the British Banking Association that reflects the interest rate at which banks in the Interbank London market will borrow yen.

The Yen TIBOR is the Tokyo Interbank Offer Rate determined by the Federation of Bankers Associations of Japan (Zenginkyo) and defining the interest rate at which banks in Japan borrow yen in the Japanese Interbank market.

Japanese Yen TIBOR is calculated using an Act/365 day basis and it is primarily used for domestic purposes.

Euro Yen TIBOR is calculated using an Act/360 basis and is composed of twelve terms (from 1-month to 12-month). The fixing panel consists of 18 banks out of which 16 are Japanese. The Zenginkyo has the right to designate these reference banks and change the number of them if necessary according to their financial activities in the Japan Offshore Market (JOM) etc. Rates are quoted by the designated banks as being, in their view, the offered rate at which Euro Yen deposits are being quoted on 360-day basis between prime banks in the JOM.

Calculation of the rates is based on the elimination of the 2 highest and 2 lowest rates from quotations and taking the average of the remainder. In the case when any bank does not indicate its offer rates, the calculation will be made in the same manner from the quoting banks.

If, everything else being the same, TIBOR is higher than yen LIBOR, then there exist a risk premium associated with default risk of the respective panel member banks.

3. **How are LIBOR, TIBOR and EURIBOR determined?**

**LIBOR** stands for London InterBank Offered rate and is the rate of interest at which banks offer funds to other banks, in marketable size, in the London interbank market. It is the primary benchmark used by banks, securities houses and investors to fix the cost of borrowing in the money, derivatives and capital markets around the world. LIBOR refers to any of a number of short-term indicative interest rates compiled by the British Bankers Association (BBA) at 11:00 AM London time, each business day. LIBOR is quoted for one-week, two-week and monthly
maturities up to a year for many of the world’s currencies (Euro, US Dollar, GB Pound, Yen, Swiss Franc, Canadian Dollar, and Australian Dollar), as well as spot/next (but overnight for EUR, GBP, USD and CAD).

All currencies are fixed on a spot basis on each London Business Day apart from GBP, which is fixed for same day value.

LIBOR fixing evolved in the early 1980’s with the growth of syndicated lending and early developments in the derivative markets. Since, it has assumed an increasing importance. it is generally acknowledged as a truly international benchmark. BBA LIBOR is published simultaneously on more than 300,000 screens throughout the world.

The number of contributing banks, quotation basis and fixing basis for each currency is given in the following table.

<table>
<thead>
<tr>
<th>Currency</th>
<th>Number of Contributors</th>
<th>Quotation basis</th>
<th>Fixing basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUD</td>
<td>8</td>
<td>a/360</td>
<td>spot</td>
</tr>
<tr>
<td>CAD</td>
<td>12</td>
<td>a/360</td>
<td>spot</td>
</tr>
<tr>
<td>CHF</td>
<td>12</td>
<td>a/360</td>
<td>spot</td>
</tr>
<tr>
<td>EUR</td>
<td>16</td>
<td>a/360</td>
<td>spot</td>
</tr>
<tr>
<td>GBP</td>
<td>16</td>
<td>a/365</td>
<td>same day</td>
</tr>
<tr>
<td>JPY</td>
<td>16</td>
<td>a/360</td>
<td>spot</td>
</tr>
<tr>
<td>USD</td>
<td>16</td>
<td>a/360</td>
<td>spot</td>
</tr>
</tbody>
</table>

The Euro BBA LIBOR Panel banks are chosen on the basis of market activity, perceived market reputation and expertise in the particular currency and are surveyed for their views of the market rate. Each bank contributes the rate at which it could borrow funds, by asking for and then accepting inter-bank offers in reasonable market size just prior to 11 AM. All Contributor Panel bank inputs are published on-screen to ensure transparency. Contributed rates are then ranked in order and only the middle two quartiles (50% of contributed rates) are averaged arithmetically.

The resulting Contributor Panel membership is supposed to reflect the international composition of the London market and the substantial trading in European currencies undertaken by banks based outside the euro-zone.
Rates similar to LIBOR are quoted in other world markets. For example, **EURIBOR** stands for Euro InterBank Offered Rate. **EURIBOR** interest rates are compiled by the European Banking Federation (FBE—Fédération Bancaire de l’Union Européenne) and are released at 11:00 AM Brussels time, each business day. Rates are quoted for one-week and monthly maturities up to a year. EURIBOR is widely used as the underlying interest rate for Euro-denominated derivative contracts.

Unlike Euro BBA LIBOR, EURIBOR fixing is based on a concept of country quota. Each in-zone country has at least one bank represented on the Panel and smaller countries will rotate membership of the Panel amongst their leading commercial banks every 6 months. EURIBOR has a panel of 57 reference banks: 47 from in-zone countries, 4 pre-in banks (HSBC and Barclays are the UK representatives) as well as 6 international banks (Bank of Tokyo-Mitsubishi, Chase, Citibank, JP Morgan Bank of America and UBS). Every panel bank is required to directly input their data no later than 10:45 a.m. (CET) on each day that the Trans-European Automated Real-Time Gross-Settlement Express Transfer system (TARGET) is open.

The averaging method of EURIBOR is to discard the top and bottom 15% of contributed rates and average the remainder.
Chapter 5

Introduction to Swap Engineering

Exercises

Question 1

1. The cash flows of this coupon bond can be separated into 8 separate payments. The first 7 of these will pay $4 at \( t_i, i = 1, \ldots, 7 \). The last payment will be of size $104. This can be used to create the synthetic:

\[
\text{Coupon Bond} = \left[ \sum_{i=1}^{7} 4B(t_0, t_i) \right] + 104B(t_0, t_8)
\]

Where the \( B(t_0, t_i) \) are the time \( t_0 \) value of the default-free discount bonds maturing at \( t_i \). These bonds pay $1 at maturity.

Hence the price of the coupon bond should equal the value on the right hand side plus a profit margin.

2. In this question the \( B_i \) are measured at annual frequencies. However, the underlying cash flows are semi-annual. Hence some sort of interpolation of \( B_i \) is needed. Using a linear interpolation, and then applying the above equality twice we can get the bid-ask prices. For example, the Bid price of the coupon bond can be calculated as:

\[
\text{Bid} = 4(.95 + .90 + .885 + .87 + .845 + .82 + .81) + 104(.80)
\]

\[= 107.52\]

According to this the coupon bond sells at a premium. This is not very surprising since, the 8% coupon is significantly higher than the annual rate implied by the term structure.

3. Here one can use either the interpolated data or the original term structure, depending on how one interprets the numbers in 1x2 FRA. Taking
these as the FRA rate on an 12-month loan that will be made in one year we get the equation:

$$1 + f(t_o, t_2, t_4) = \frac{B(t_o, t_2)}{B(t_o, t_4)}$$

Replacing from the term structure we obtain:

$$f(t_o, t_2, t_4)^{Bid} = \frac{0.90}{0.88} - 1$$

$$= 2.27\%$$

**Question 2**

1. Note that as Italian Government buys back 30-year bonds, the sovereign curve will shift “down”, relatively more than the swap curve. Or, the sovereign curve will shift up, less than the swap curve, depending on the direction of the movement. The typical investor is receiving fixed 30-year government yield and paying fixed in the swap market. There is also the spread received over Euribor. Such an investor will realize capital gains if yield curve movements occur as expected.

To see this, note that one can approximate the value, $V_t$, of the position described in the second paragraph using:

$$V_{t_o} = \sum_{i=1}^{30} (R_{t_o} - (R_{t_o} + s_{t_o}) + 0.0105) B(t_o, t_i) + 100$$

where we assume, unrealistically, that the $t_i$ run over years. The $R_t, s_t$ are the sovereign yield, and swap spread respectively. (This is approximate, since we are applying the same discount factors to Euribor-based payments and the sovereign interest payments. Normally, these are related to different discounts.) The Libor-based payments have the present value of 100, which is shown as the last term on the right hand side. It is for this reason that no $L_{t_i}$ appear on the right hand side.

Now, if the Italian government buys back the 30-year bonds, we expect the $B(t, t_i)$ to increase. Ceteris Paribus, this will leave the $s_t$ unchanged and the position will gain. If the spread over Euribor falls in addition to this, the gains will be even higher.
2. Yes, the trade in the reading will benefit most if 30-year bonds are repurchased.

3. This is the standard cash flow diagram. The implied graph will be similar to parts (a) and (b) of Figure 5-7.

4. Ignoring any differences in day counts and other conventions, this makes the swap rate equal \(0.06 - 0.00105\) if paid against Euribor flat.

5. Investors enter more of such positions and the swap rate will increase due to supply-demand, this is equivalent to a decrease in the spread over Euribor.

**Question 3**

1. You will draw three separate cash flow diagrams, each representing a different swap. Assume for simplicity, that the swaps are against 12-month Libor. Remember that the notional amounts are different.

2. For the net payments, one can simply add vertically the cash flows at each \(t_i\).

3. Once these cash flows are determined for each \(t_i\), one would multiply them with the appropriate discount factors, obtained from the swap curve.

   For example, the cash flows two years later, at \(t_2\), will be given approximately by:

   \[
   \frac{(0.0675 - 0.0675)(50m) + (0.07 - 0.0688)(10m) - (0.0755 - 0.0745)(10m)}{(1 + 0.0675)^2}
   \]

   The point to remember is that, the unknown Libor-based payments can be replaced by the corresponding values measured using the current swap rate given in the Table.

   Note that the Swap curve starts at year 2 and some form of interpolation is needed for the first year cash flows. Alternatively a Libor curve will be needed.

4. This will be positive as the above example shows.
5. One could hedge the net position that has a five year maturity with a 4-year swap only in an approximate sense. One would calculate the durations of the two swaps and then take a position that equates the first order sensitivities of the net position and the hedge.

6. An exact hedge can be put together by entering into 5 different FRA contracts.

Question 4

1. The question implies that the \( t_i \) are measured at annual intervals. Using this we can easily calculate arbitrage-free bid-ask prices for the two zero coupon bonds. For example,

\[
B_{2}^{\text{ask}} = \frac{1}{(1 + .0810)(1 + .0901)} = .8486
\]

\[
B_{2}^{\text{bid}} = \frac{1}{(1 + .0812)(1 + .0903)} = .8483
\]

Note that there is a small arbitrage possibility. One can buy the synthetic bond at .8486 and sell the actual bond for .85 at the quoted price. This will leave a gain of .0014.

2. Three period swap rate will be a weighted average of the quoted forward rates:

\[
s_0 = \sum_{i=0}^{2} \omega_i f_i
\]

where the \( \omega_i \) are given by:

\[
\omega_i = \frac{B_{i+1}}{B_1 + B_2 + B_3}
\]

The key point in applying this formula is the following. Instead of using the \( B_i \) quoted by the dealer, one needs to calculate arbitrage-free bond prices as in part (a) of this question.

30
Question 5
Additional data on USD and EUR FRA’s will not be directly relevant for finding arbitrage opportunities in the GBP sector. However, they will be relevant if one had in addition, quotes on forward GBP/USD and forward GBP/EUR exchange rates. Such forward rates incorporate not one, but two term structures and the additional data might help.

Question 6
1. The foreign investor is subject to withholding tax if the issuer is a resident of Australia. In this case the question suggest that the Spanish issuer is not a resident institution. So, the foreign buyer is not subject to withholding taxes.

2. A resident issuer who would like to issue AUD bonds, can issue in a different currency and then swap the proceeds with a foreign issuer. In this case the foreign issuer is issuing in AUD and swapping to the other currency. Hence there will be two fixed rate swaps and a currency swap that will have to be involved.

3. With the data provided in this question no precise numerical answer can be given. However, the arbitrage gains will be within 10% of the quoted rates.

4. FRA’s themselves are not sufficient. One would need in addition the proper forward exchange rate contracts on, say, the USD/AUD. One would also need the USD FRAs. Then one can synthetically reconstruct all the IRSs and the currency swaps desired.

5. Theoretically they will give the same results. However, FRAs will involve a much larger number of contracts and may end up being less convenient.

6. The Spanish company will issue in Australia and then swap the proceeds into a desired currency. This will be more “profitable”.

Question 7
1. This is similar to Figure 5-6.
2. The bottom part of Figure 5-6 shows this.

3. Here we should first note that the currency swap spreads of around 75 basis points are unrealistically high for real markets. For these currencies such spreads were normally around 10-20bp during 2004.

Ignoring this aspect we can see that for the immediate period issuing in EUR and then swapping the proceeds into USD will yield an all-in cost that is about 10bp higher for the immediate settlement period, since the issuer will be paying 5.8% in USD after the currency swap.

This issuer will pay USD Libor-90 and then receive EUR Libor flat.
Chapter 6
Repo Market Strategies in Financial Engineering
Exercises

Question 1
1. 0% haircut implies the collateral is 30 million EUR and the cash received is also 30 million.

2. Now the Bund is more valuable than previously thought: by $101 - 100.50 = 0.50$. So, a portion of the bonds or equivalent amount of cash must be returned to the dealer by the borrower of the bond.

3. 2.7%.

Question 2
1. 5% haircut implies the “lender” (of the bond) receives $0.95 \times 10,000,000 = 9,500,000$ dollars

2. The dealer earns interest at 2.5% on the $9.5$ million he lends. But, the dealer can also re-lend 5% of the borrowed securities with 0% haircut and earn extra interest.

Question 3
1. Dirty price is the clean price plus accrued interest $= 97 + 3 = 100$

2. On the dirty price.

3. Dollars received

\[
= 0.97 \times \frac{1}{0.87} \times 40m = 44.59m
\]

(taking into account the exchange rate and the haircut)

4. 3%.
Chapter 6

Repo Market Strategies in Financial Engineering

Case Study: CTD and Repo Arbitrage

2 Definitions

1. General collateral (GC)
   
   This is the case of a repo transaction between two parties where the borrower is willing to receive as collateral any of the securities satisfying a general set of criteria.

   The GC rate will be higher than the special Repo rate.

2. Special Repo
   
   The borrower asks for a particular security as collateral. Other similar securities are not accepted.

3. Cheapest to Deliver (CTD)
   
   Bond futures are structured so as to result in physical settlement. The short who decides to deliver, will have to deliver a physical bond. But, usually, bonds of a certain maturity do not have large sizes to accommodate quick deliveries. Hence, the futures contracts will trade a hypothetical contract and then a physical bond that resembles to the theoretical bond is delivered. For this reason Exchanges designate a set of bonds as belonging to a deliverable basket. Although the amount of each bond to be delivered is adjusted\(^1\) so that they become similar to the settlement value of the theoretical bond at expiration, one of these deliverable bonds will in general be cheapest to deliver. The shorts will obviously deliver this bond called the CTD instead of more expensive ones to satisfy their liability.

   In this case study, the CTD is the 6.5% Bobl (see below) with maturity October 2005.

\(^{1}\)This is done through the so-called adjustment factor.
4. Failure to Deliver

This occurs when a repo or futures counterparty that is short the bond, fails to deliver it will result in a failure to deliver. There are daily penalties for failure to deliver.

3 Consequences

The fees for failing to deliver in repo markets are 1.33 basis points, whereas the fees for failing in Futures markets is 40bp. Hence, an agent who is short in the futures market will be penalized much more than an agent who is short in the repo market. This means that the shorts in the futures markets may be more willing to deliver the more expensive bonds instead, if they cannot find the CTD in the open market.

The strategy works better if the profit from more accepting delivery of more expensive bonds is greater than the 1.33 basis points DB will be paying as penalty in the repo market.

The position of the DB will be as follows:

1. Let $B(t_o, T)$ denote the price of the CTD at time $t_o$. DB will place this amount with the repo dealer at the repo rate $r_{t_o}$. The second leg of repo is chosen by DB so as to settle at time $t_1 - \Delta$. Where $\Delta$ is a short period of one or two days. On this date, the exchange of cash and bond is reversed and DB receives the repo interest.

2. Assume that DB borrows the cash placed with the repo dealer at the going Libor rate $L_{t_o}$.

3. Ignore the mark-to-market in the futures position. The futures position does not involve cash at time $t_o$. It is assumed to settle at $t_1$. Note that according to this, the CTD bonds are supposed to be returned to the repo dealer before futures contract expires. Hence, at that point these bonds will be available in the repo market.

4. Yet, if DB fails to deliver, these bonds will not be available to the shorts.

Let’s now see how this can result in a squeeze.
3.1 What is a Squeeze?

According to these positions, DB is long the March 2001 contract. This will put upward pressure on the futures price $F_{t_1}$. But DB is also borrowing the underlying CTD in the repo market.

Although, originally the bonds are supposed to be returned to the repo dealer before the $t_1$, DB fails to deliver in the repo market. This removes these bonds from the repo market for a few days.

Then, the shorts are either forced to close their positions by offsetting them with new long positions, which means higher futures prices, or, by delivering the more expensive bonds from the deliverable basket. Either way, DB profits.

The following excerpt is from the BIS Quarterly Review, June 2001. It deals with the squeeze discussed in this case study. (See, Anatomy of a Squeeze by Serge Jeanneau and Robert Scott in the above-mentioned BIS Review.)

The remarkable success of German government bond contracts has created some difficulties in recent years. Most recently, a market squeeze on the bobl contract was reported during the first quarter of 2001. The bobl is the five-year German government note, which is used as the underlying asset for related futures and options traded on Eurex. A small number of European banks apparently cornered the cheapest-to-deliver (CTD) note for the contract maturing in March 2001, causing major losses to traders with short positions.

In futures markets, squeezes occur when holders of short positions cannot acquire or borrow the securities required for delivery under the terms of a contract. Delivery does not normally pose a problem for traders because the majority close their positions with offsetting transactions prior to contract expiry. However, a trader who remains short at the contracts expiration is obliged to deliver the specified securities, just as one who remains long must take delivery.

Because of the difficulty in obtaining transparent prices in bond markets, most contracts on government bonds require physical delivery. This is in contrast to contracts on interbank rates and
equity indices, which are settled in cash on the basis of transparent price indices. Physical delivery requires specification of the range of eligible securities and a pricing mechanism to turn the different securities into equivalent assets.

(...)

In the case of the bobl future, the deliverable securities are German government notes with maturities between 4.5 and 5.5 years. To adjust for differences in coupons and maturities, the prices of these bonds are multiplied by a conversion factor based on a valuation of coupons and principal at an annual yield of 6 dates. However, because this adjustment is imperfect, one of the securities will always turn out to be cheapest to deliver, depending on the level of market interest rates and the slope of the yield curve.

(...)

Squeezes are more likely if the supply of the CTD is small, if the choice of CTD is highly predictable and if its rotation to other deliverable securities is prevented by a lack of issues with fairly similar price sensitivities.

(...)

Market circumstances in February 2001 appear to have provided a good opportunity for a squeeze. The CTD was the 6.5% note maturing in October 2005. Open interest in the bobl future rose to over 565,000 contracts by 22 February, amounting to a notional amount of 57 billion. This was over five times the stock of CTD notes and about one and a half times the total size of the deliverable basket. By contrast, the December and September 2000 contracts had respectively only 384,000 and 281,000 futures outstanding two weeks before expiry.

The graph below illustrates the dynamics of this squeeze. We see that open interest of the March 2001 had increased significantly. At the same time, we see that the spread between the yield of the CTD and the yield of the next cheapest bond went down from 3 bp to negative 2 bp. This means that the original CTD became more expensive as the expiration of the contract approached.
The squeeze in the March 2001 contract (BIS Quarterly Review, June 2001)

Figure 5:
4 Hedging the short position

Can shorts use other ways to hedge their risks? Consider first a Total Return Swap. A Total Return Swap (TRS) is an exchange of interest and capital gains/(losses) generated by a fixed income security against Libor plus a spread.

According to this in a TRS swap the receiver gains exposure to the underlying bond without any capital and without physically owning the asset.

Clearly, the shorts could hedge their short positions in the CTD bond by entering a TRS. The TRS has to be set up with that particular bond as the underlying. Of course, the spreads to Libor may already incorporate the price movements due to the squeeze and the hedge may not, at the end, be very useful. This especially is the case here, since the counterparty to the TRS deal will have to hedge his or her position and this means getting the CTD in the open market. This party will find out that this particular bond is expensive.

On the other hand, the use of FRA’s will not help the shorts. It is unlikely that the FRA will move significantly due to this squeeze. Squeezes involve CTD’s that are in relative small sizes when originally issued. A shortage in a small issue will not affect the overall level of bond prices.

4.1 Zero repo rates

When a security is heavily demanded by market participants, the repo dealers will lend it at an “expensive price”. The way this occurs is by adjusting the repo rate.

Hence, the CTD in this question will be in heavy demand. To receive this bond in the repo market, shorts will have to surrender their cash at a rate lower than the normal repo rate. At the extreme, the repo rate will approach zero.

On the other hand, the repo dealer is securing funds at a cost of 0% while lending them with a return of $r_{t_1}$.

5 Other Readings

There are some other readings on this particular event that the readers may find useful. One is “The Banker, October 2001, Traders Squeeze Bobl”. Another is BID quarterly Reviews, for example, June 2002 issue.
An academic paper that helps to understand the mechanics of squeezes and the related dynamics is by John Merrick, Narayan Naik and Pradeep Yadav, et. al. “Strategic Trading Behavior and Price distortion in a Manipulated Market: anatomy of a Squeeze.”
Chapter 7

Dynamic Replication Methods and Synthetics

Exercises

Question 1

1. The option has a maturity of 200 days. In order to have 5 steps, the $\Delta$ must correspond to 40 days. But, we let a year denoted by 1. In this case, using a days convention of act/365, the $\Delta$ becomes:

$$\Delta = \frac{40}{365}$$

2. In order to answer this question we need further assumptions about the tree structure. The question implies that the probability of the state denoted as “up” is constant. To obtain this probability we need to discretize the stochastic differential equation.

One way to proceed is the following. We discretize the continuous time dynamics using the Euler scheme and replace the $\mu$ with the risk-free rate. We obtain:

$$\Delta S_{t_i} \approx [0.06\Delta + \sigma\epsilon_{t_i}]S_{t_{i-1}}$$

where $\epsilon$ is a two-state random variable, states being denoted by “up” and “down” respectively. We have the following two-state dynamics for $S_t$:

$$S_{t_i}^{up} = uS_{t_{i-1}}$$

$$S_{t_i}^{down} = dS_{t_{i-1}}$$

where

$$u_i = 1 + 0.06\Delta + \sigma\epsilon_{up}^i$$

Let $p$ be the probability of down state. And assume, a usual that

$$E_{t_{i-1}}[\epsilon] = 0$$

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Then we have the following equations:

\[ \epsilon_{up} p + \epsilon_{down} (1 - p) = 0 \]

which is equivalent to:

\[ (uS_{t-1})p + (dS_{t-1})(1 - p) = (1 + .06\Delta)S_{t-1} \]

where the \( S_{t-1} \) can be eliminated. At this point we have three unknowns, \( p, \epsilon_{up}, \epsilon_{down} \). We need more conditions.

We can let the three to recombine, so that an “up” movement followed by a “down” movement results in the same value as a “down” followed by an “up”:

\[ ud = 1 \]

We can now solve for \( p, \epsilon_{up} \) or \( p, \epsilon_{down} \). Letting \( \epsilon_{up} = 2\Delta \) gives,

\[ p = .663 \]
\[ \epsilon_{down} = -1.970\Delta. \]

Note that the values of the random up and down movements, \( \epsilon^i \) should depend on the \( \Delta \). This is needed since, the variance of this term will have to be proportional to \( \Delta \). This is one consequence of \( W_t \) being a Wiener process, in the continuous form of the dynamics.

**Question 2**

1. In this case we would follow the same steps as in Exercise 1, after replacing the drift of \(.06\Delta \) by \((.06 - .04)\Delta\).

2. The tree will be the same as in Exercise 1, until the third step. Then, all values are lowered by 5%.

3. The tree will shift down by 5$ after the third step. This will complicate the tree since, after the third step the remainder of the tree will become non-recombining. The reason is that a constant dollar sum is deducted from the relevant \( S_t \) and this will correspond to a slightly different percentage change for the “up” and “down” states.
Question 3

We will solve the problem using the GBP/USD exchange rate of $1.85 USD interest rates of 1.5% and GBP interest rates of 5.5%

1. The new drift, which will be \((0.015 - 0.055)\Delta\) does not affect the choice of \(\Delta\). We still have \(\Delta = 40/365\).

2. Using the same method as in the answer for question 1, we find:

\[
p = .64 \\
\epsilon^{up} = 1 \\
\epsilon^{down} = -1.7789
\]

which gives

\[
u = 1.0154 \\
d = .985
\]

3. Using the initial point of \(S_1 = 1.85\) recursively calculate:

\[
S^up_i = uS_{i-1} \\
S^{down}_i = dS_{i-1}
\]

4. In order to calculate the tree for the European Put we start from the last step, \(i = 5\). Given that the exchange rate tree is recombining we will have a recombining tree for the Put option. there will be 5 nodes at step \(i = 5\). These values will be

\[
\{u^2S, uS, S, dS, d^2S\}
\]

Using the \(u, d\) found in the previous question we can calculate the values of the exchange rate at step \(i = 5\) and find the values of the put option denoted by \(P_i\) by letting:

\[
P_j^5 = max[1.50 - S_j^5, 0]
\]

where \(j = 1, \ldots, 5\) represent the 5 possible states of the last step. We can then work backwards, using the relation:

\[
P_{i-1} = \frac{1}{1 + .04\Delta}[pP^up_i + (1 - p)P^{down}_i]
\]
5. The American put requires working backwards in the tree as done in the previous question. However, at each node we need to check one more condition. The option holder can, at each node exercise the option. Thus the value of the relevant $P_i$ should be compared with $1.50 - S_i$. If the latter is greater, then the option will be exercised.

This means that the subsequent values on the tree will have to be let equal to zero. Also, this exercise value will be discounted properly in obtaining the value of $P_1$.

Question 4

1. The existence of many Put writers suggest that, these players’ positions will lose money when the market drops. To hedge this position, they have to sell short the underlying stock in the right amount.

This can be shown by using a standard short Put payoff diagram.

2. A covered put position means, the trader has written a put and then shorted the underlying by selling $delta$ units of the underlying.

Now suppose the market drops further. the trader is hedged against this. But, the drop of the market implies a bigger $delta$ in absolute value. So, more of the underlying needs to be shorted. Note that this means further sales hitting the market. It is this point that is being refuted by the traders in the first paragraph of the reading.

3. Short volatility positions are created by selling options, among others. The same trader can be long volatility somewhere else if they either own similar options or if they are long convexity.

4. This is quite possible. Note that cash markets are much smaller than the markets for derivatives on the same underlying. Thus the derivative market may be marginally short, but this may lead to significant short sales in the corresponding cash market.

5. The last paragraph of the reading suggests that most players try to cover their volatility positions. This may reduce the need for adjusting the corresponding delta hedge.
Question 5

In order to check whether or not the trees in Figure 7-7 are arbitrage free we would check whether the equality,

\[ S_{i-1} = \frac{1}{1 + r^j \Delta} [S_{i}^{up} p + S_{i}^{down} (1 - p)] \]

is satisfied at every node.
Chapter 8
Mechanics of Options
Exercises

Question 1

1. The payoff diagrams will look as in Figure 1.

2. Gross payoff at expiry will be:

\[ P(T) = -\min[(1.23 - S_T), 0] + \min[(1.10 - S_T), 0] \]

where \( S_T \) is the EUR/USD exchange rate at expiration.

3. The net payoff will be given by:

\[ P(T) - P_{1.23} + P_{1.10} \]

where \( P_{1.23}, P_{1.10} \) are the premiums of the corresponding options.

4. If there is a volatility smile, then the implied volatility of the out-of-the-money put will be higher than the implied volatility of the ATM Put. The trader selling the out-of-the-money volatility and buying the ATM volatility. Hence if the “smile” flattens, the trade gains.

Question 2

1. Long gamma means, buying related Puts and/or Calls and then delta hedging these positions with the reverse position in the underlying. The hedge ratio will be gamma. This isolates the convexity of option payoffs and benefits from increased volatility.

If markets have not priced-in the increase in volatility that may result from (anticipations of ) FED announcements, then the trade will benefit. Realized volatility will be higher than the volatility priced in the options. Gamma gains would exceed any interest expense and time decay during the 7 day period.
2. Here we can calculate the gamma of at-the-money options. We can assume interest rate differentials around 3%. We can let the life of the option be 7 days.

Such ATM options would have maximum gamma, since the price curve will be very close to the piecewise linear option payoff diagram. This means that the traders are maximizing their exposure to increased volatility through Gamma.

3. Given the volatility, we can approximately calculate possible gains by letting

\[ N \frac{\partial^2 C}{\partial e_t^2} e_t \frac{7}{365} (\sigma^2_{\text{realized}} - \sigma^2) \]

where \( e_t \) is the expected USD/NZD exchange rate and \( N \) is the notional amount which is said to be around USD10-20 millions.

Calculating the Black-Scholes Gamma and then plugging in the relevant quantities in the above formula will give approximate size of expected gains for various realized volatilities.

**Question 3**

1. Buying sort-dated euro Puts and the implied Gamma means that traders will go long EUR/USD exchange rates. Thus they will buy Euro and sell Dollars.

2. Buying euro puts is a hedge for further drops in euro.

3. Triggering of barrier options may lead to relatively large movements. This may or may not increase the realized volatility. If it does, then buying Gamma will be the natural response.

**Question 4**

1. Two very crude approximations for Delta are,\n
\[ \frac{C(S + \Delta S) - C(S)}{\Delta S} \]
\[-C(S - \Delta S) + C(S) \over \Delta S\]

A better approximation is

\[C(S + \Delta S) - C(S - \Delta S) \over 2\Delta S\]

In fact, applying these to the data shown in the Table we see that only in the last case we obtain an ATM delta of around .5. The two other cases give very different ATM Deltas.

2. ATM delta is around .5 as the third method illustrates. We can similarly calculate the Delta for spot equal to 25. However, we cannot use the third method when \(S = 10\), or when \(S = 30\). For these the first and the second formula need to be used.

Once these deltas are calculated, then we can calculate daily gains/losses as:

\[-r1.3 \over 365 + {1 \over 2} [Delta_t - Delta_{t-1}][S_t - S_{t-1}] + Time - decay\]

3. In this case the volatility is much higher and the Gamma gains will be higher as well.

**Question 5**

1. Volga is a Greek relevant for Vega hedging. It is the second derivative of the option price relative to the volatility parameter,

\[Volga = \frac{\partial^2 C}{\partial \sigma^2}\]

2. Vanna represent the derivative of the Vega with respect to the spot price,

\[Vanna = \frac{\partial^2 C}{\partial \sigma \partial S_t}\]

3. These Greeks can be considered as changes in Vega when volatility and the underlying spot price change. Hence they will be relevant for hedging and measuring Vega exposures.
At-the-money put

Figure 6:
Chapter 9

Engineering Convexity Positions

Exercises

Question 1

1. By definition, the price of a coupon bond will be given by,

\[
P(0, t_n) = \left[ \sum_{i=1}^{n} \frac{c}{\prod_{j=1}^{i} (1 + y)^j} \right] + \frac{100}{\prod_{j=1}^{n} (1 + y)^j}
\]

Since the yield curve is flat, \( y_k = y_1 \) for all \( k \). The forward rate, \( f_i \), is defined as

\[f_i = \frac{\prod_{j=1}^{i} (1 + y)^j}{\prod_{j=1}^{i-1} (1 + y)^j} - 1 = y.\]
2. \( P(0,30) = $87.59, y = 7\% \). Bond delta, using the price equation in part a, can be expressed as:

\[
\frac{dP}{dy} = -\frac{c}{y} \left[1 - (1 + y)^{(-n)}\right] + \frac{c}{y} \times \frac{n}{(1+y)^{n+1}} - \frac{100n}{(1+y)^{n+1}}
\]

where \( n \) is years to maturity, and \( c \) is the coupon payment. So initially, bond delta is \(-1116\) and forward contract delta is \(-100\). In order to be delta neutral, we must short 11 forward contracts for each long coupon bond in the portfolio.

In order to construct a zero cost portfolio, we need, in addition, to borrow $87.59 at the ongoing interest rate 7\%. (Remember that value of the forward contract is initially zero.)

According to this result, if we want to construct a zero cost portfolio, we need to borrow $87.59 at the ongoing interest rate (7\%) (Remember that value of the forward contract is initially zero).

3. See the table below. Sum of the entries on the last column is the total convexity gain.

<table>
<thead>
<tr>
<th>Yield</th>
<th>Bond Delta</th>
<th>Forward Delta</th>
<th># of Forwards</th>
<th>Price</th>
<th>Mark-to-Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>-1116</td>
<td>-100</td>
<td>11</td>
<td>93</td>
<td>0</td>
</tr>
<tr>
<td>0.09</td>
<td>-754</td>
<td>-100</td>
<td>8</td>
<td>91</td>
<td>22</td>
</tr>
<tr>
<td>0.07</td>
<td>-1116</td>
<td>-100</td>
<td>11</td>
<td>93</td>
<td>-15</td>
</tr>
<tr>
<td>0.09</td>
<td>-754</td>
<td>-100</td>
<td>8</td>
<td>91</td>
<td>22</td>
</tr>
<tr>
<td>0.07</td>
<td>-1116</td>
<td>-100</td>
<td>11</td>
<td>93</td>
<td>-15</td>
</tr>
<tr>
<td>0.09</td>
<td>-754</td>
<td>-100</td>
<td>8</td>
<td>91</td>
<td>22</td>
</tr>
<tr>
<td>0.07</td>
<td>-1116</td>
<td>-100</td>
<td>11</td>
<td>93</td>
<td>-15</td>
</tr>
</tbody>
</table>

\( Gains: 22 \)
4. Other costs are funding cost and other operational costs, fees and commission paid.

**Question 2**

1. Price of 30 year bond is,

\[ B(0, 30) = \frac{100}{(1 + y)^{30}} \]

and it is equal to $23.14 when \( y = 5\% \). In order to meet the zero-cost condition, we borrow $23.14 at a rate of 5%. Bond’s delta is given by,

\[ \frac{dB}{dy} = -\frac{3000}{(1 + y)^{31}} \]

So initial bond delta is \(-661\) and euro dollar contract delta is \(-25\). That means for each long bond position, we must short \( \frac{661}{25} \) euro contracts to achieve delta neutrality, initially.

2. The solution to this problem is very similar to solution given for question 1, part (d) above. The sum of the entries on the last column is the convexity gains.

<table>
<thead>
<tr>
<th>Yield</th>
<th>Bond Delta</th>
<th>ED Delta</th>
<th># of Contracts</th>
<th>Price of ED</th>
<th>Mark-to-market</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>-661</td>
<td>-25</td>
<td>26</td>
<td>98.75</td>
<td>0</td>
</tr>
<tr>
<td>0.06</td>
<td>-493</td>
<td>-25</td>
<td>19</td>
<td>98.5</td>
<td>6.5</td>
</tr>
<tr>
<td>0.04</td>
<td>-889</td>
<td>-25</td>
<td>35</td>
<td>99</td>
<td>-9.5</td>
</tr>
<tr>
<td>0.06</td>
<td>-493</td>
<td>-25</td>
<td>19</td>
<td>98.5</td>
<td>17.5</td>
</tr>
<tr>
<td>0.04</td>
<td>-889</td>
<td>-25</td>
<td>35</td>
<td>99</td>
<td>-9.5</td>
</tr>
<tr>
<td>0.06</td>
<td>-493</td>
<td>-25</td>
<td>19</td>
<td>98.5</td>
<td>17.5</td>
</tr>
<tr>
<td>0.04</td>
<td>-889</td>
<td>-25</td>
<td>35</td>
<td>99</td>
<td>-9.5</td>
</tr>
</tbody>
</table>

*Convexity gains: 13.00*
Question 3

Interest rate fluctuations are wider in this question compared to previous question. That means higher volatility which implies higher convexity gains. So, the total rate of return on this bond will be higher, while interest rate for 30 year bond decreases.

Question 4

Let $f$ be forward rate on Libor-on-arrears FRA. And $F$ be the forward rate on market traded FRA. Then, existence of convexity requires the following adjustment between these two forward rates:

$$f = F + \sigma^2$$

So the spread is equal to $\sigma^2$, $(0.02)^2$. 
1. We now explain the notion of convexity of long bonds.

For a given decrease in the yield, bond prices will increase more compared to the linear approximation of the price-yield relationship. For a given increase in the yield, bond prices will decrease less compared to the linear approximation of the price-yield relationship.

The Figure 1 above shows this.

We can also give an example.

Let us find the bond sensitivities for a 3-year bond and a 30-year bond given the following conditions:

(a) \( y_o = 6.5\% \) and \( \Delta y_o = 0.3 \)
(b) \( y_1 = 6.9\% \) and \( \Delta y_1 = 0.6 \)

where, \( y_o, y_1 \) are the yields of 3 year and 30 year default-free discount bonds, whose prices are denoted by \( B_3 \) and \( B_{30} \). These prices will be
given by:

\[ B_3 = \frac{100}{(1 + y_o)^3} \]

\[ B_{30} = \frac{100}{(1 + y_{10})^{30}} \]

The first order sensitivities are related to these bonds duration. For the short bond this will be given by:

\[
\frac{\partial B_3}{\partial y_o} B_3 = (-3) \frac{100}{(1 + y_o)^{3+1}}
\]

\[
= \frac{-300}{(1 + y_o)^4}
\]

\[
= 3 \left( \frac{1}{(1 + y_o) (1 + y_o)^3} \right)
\]

\[
= 3 \left( \frac{1}{(1 + y_o) B_3} \right)
\]

This means that the percentage in the bond price will be:

\[
\frac{\partial B_3}{\partial y_o} = (-3) \frac{100}{(1 + y_o)^{3+1}}
\]

\[
= \frac{-300}{(1 + y_o)^4}
\]

\[
= 3 \left( \frac{1}{(1 + y_o) (1 + y_o)^3} \right)
\]

\[
= 3 \left( \frac{1}{(1 + y_o) B_3} \right)
\]

\[
\frac{\partial B_3}{\partial y_o} B_3 = 3 \left( \frac{1}{(1 + y_o)} \right)
\]

Hence the term modified duration. The right hand side in this expression gives the slightly modified maturity of the cash payments associated with this security.
For the long bond we get:

$$\frac{\partial B_{30}}{\partial y_1} \frac{1}{B_{30}} = 30 \frac{1}{(1 + y_1)}$$

We can use this in order to get approximate measures of bond price sensitivities. For example:

$$\frac{\Delta B_3}{B_3} \approx 3 \frac{1}{(1 + y_o)} \Delta y_o$$

$$\frac{\Delta B_{30}}{B_{30}} \approx 30 \frac{1}{(1 + y_1)} \Delta y_1$$

These measures indicate that, the 30-year bond will be about 10 times more sensitive to an interest rate change than a 3-year bond, for the same amount of yield movement.

We can also calculate second order sensitivities. These convexity or "Gamma" effects will show how the first order sensitivities change as the yield moves.

$$\frac{\partial^2 B_3}{\partial y_o^2} \frac{1}{B_3} = (12) \frac{1}{(1 + y)^2}$$

and

$$\frac{\partial^2 B_{30}}{\partial y_1^2} \frac{1}{B_{30}} = (930) \frac{1}{(1 + y)^2}$$

Thus, the long bond duration will be about 80 times more sensitive to changes in the yield when compared with the short bond.

2. Suppose we short the 3-year bond and go long on the 30-year bond. Then, consider three cases where interest rates move up +0.3, stay same or move down −0.3. How will all these affect the bond portfolio?
• If interest rates rise we will gain more on the short position of less convex bonds than the amount we would lose on the long position of more convex bonds;

• If interest rates fall we will gain more on the long position of more convex bonds than the amount we would lose on the short position of less convex bonds;

• If interest rates remain unchanged, portfolio’s value will remain the same.

According to this we are short the lesser convex bond and long the more convex bond. As yields fall the price of this latter rises higher than the less convex bond and as yields rise its price falls less.

3. Swap convexity will be similar to coupon-bond convexity analysis. Consider the following terminology:

(a) $s_t = \text{Swap rate at time } t \text{ on a swap that starts at } t$.

(b) $n = \text{Number of swap settlements}$.

(c) $\delta = \text{Tenor of the floating leg. } \delta = 1/4 \text{ corresponding to a 3-month floating rate leg}$.

(d) $N = \text{Notional amount}$.

(e) $L_{t_i} = \text{Libor rate to settle in-arrears at time } t_{i+1}$.

(f) $F_{t_i} = \text{The FRA rate that corresponds to the floating Libor rate } L_{t_i}$.

(g) fixed and floating day basis, both 30/360.

Under these conditions the value of the swap at time, $t_o$, will be given by:

$$V_{t_o} = \sum_{i=1}^{n} \left[ \frac{(F_{t_{i-1}} - s_{t_o})N\delta}{\prod_{j=1}^{i}(1 + F_{t_{i-1}})\delta} \right]$$

Note that this is a convex(concave) function.
Note that this gives the discounts $B_i$ as of $t_o$ as:

$$B_i = \left[ \frac{1}{\prod_{j=1}^{i} (1 + F_{t_{i-1}} \delta)} \right]$$

Now suppose we consider two swaps with $n = 3$ and $n = 5$ respectively, with $\delta = 1$. The Swap notional is 10m. The yield curve is flat at 4%. This makes all forward rates equal 4%. Then we can calculate the following numbers using the formula above.

| Changes in the  
value of the swap | Scenario 1 | Scenario 2 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Swap</td>
<td>Parallel Shift down 1%</td>
<td>Parallel shift up 1%</td>
</tr>
<tr>
<td>3 yr swap</td>
<td>- $193,887.49</td>
<td>$ 188,338.21 (value 0 at t = 0)</td>
</tr>
<tr>
<td>5 yr swap</td>
<td>- $376,497.28</td>
<td>$ 358,940.77 (value 0 at t = 0)</td>
</tr>
</tbody>
</table>

From these numbers we see that the 5-year swap is more sensitive to the changes in the interest rates than 3-year swap; therefore, the trader might gain more by trading long-dated swaps.

4. We let a Libor-in-arrears instrument pay according to the following function:

$$V_t = 100(1 - L_t \delta)$$

A eurodollar futures has this pricing function. If a position is taken at time $t_o$ with the forward rate $f_{t_o}$ the net payoff at $t_i$ will be:

$$V_{t_i} - V_{t_o} = (f_{t_o} - L_{t_i})N \delta$$

A market traded FRA on the other will have the time $t_i$ payoff

$$W_{t_i} = \frac{(F_{t_o} - L_{t_i})N \delta}{(1 + L_{t_i} \delta)}$$

Note that one payoff is Linear in $L_{t_i}$ whereas the other is non-linear.
The $\epsilon$ in the relationship between $f_{t_o}$ and $F_{t_o}$ gives the convexity adjustment:

$$F_{t_o} = f_{t_o} + \epsilon$$

Under these conditions the two forward rates would not be the same due to the convexity.

5. The position taken by the knowledgeable professionals can be summarized as follows:

(a) Receive Libor-in-arrears with a Libor-in-arrears FRA
(b) Pay Libor at the start of the period using a market traded FRA.
(c) Sell caps against the Libor-in-arrears being received.
(d) Delta hedge the swap

In this environment, swaps are more convex as they are equal to a series of FRAs.

6. Knowledgeable market professionals take their position using swaps. We discuss the answer using FRAs. Swaps can be reconstructed from FRAs and hence our approach can be duplicated for swaps as well.

Essentially, the less competent professionals are using the same $f_{t_o}$ in valuing both FRAs, the paid-in-arrears and the Libor-in-arrears.

Thus by buying the Libor in arrears FRA and selling the paid arrears FRA, one would end up with the net convexity adjustment factor $\epsilon$. This factor is equal to

$$\epsilon = F^2_{t_i} \sigma^2 \Delta \frac{1}{1 + F_{t_i} \delta}$$

7. At this point the position taken is not true arbitrage, because the gains depend on the level of volatility, although they are always positive. But, once the transaction costs are taken into account, the position may lose money if the volatility goes down significantly. This is due to the fact that the gains are a function of the $\sigma^2$.

Besides, there are the usual counterparty risks.
8. A cap is a series of European interest rate call options covering a different forward time periods each with the same strike price. You can think of it as a series of call options on FRAs. Caps are priced using Black’s formula, which assumes a forward rate that moves as a Martingale.

9. Smart Traders can lock-in their potential convexity and volatility gains by selling a cap on the forward yields. Premium from the cap includes implied volatility expectation for the remaining time to the next period as well as the expected convexity gains.
Question 1
The pay off diagram for short futures position is given in 4a:

Here, we assume that initial futures price is $5.

1. A bear spread can be created by buying a call option with strike price of $K_1$ and by selling a call option with a strike $K_2$, such that $K_1 < K_2$. Now assume that the investor does not expect a future’s price less than $3$. Under this new assumption expected pay off diagram of short future’s position would look like:
2. Now we create a similar payoff by using a bear spread:

Here, the net payoff is from the bear spread. It is clear from the figure that, if investor does not expect that underlying price will go below some minimum (here this is assumed $3), then bear spread provides the same payoff as a short future’s position. Yet, it also puts a limit on potential losses in case underlying price does go up.

How do we construct a bear spread in general? In other words, how do we choose two call options?

We short the call with a strike price \( K_1 \) such that

\[
K_1 = X_{\text{min}}.
\]

The second call is chosen so that

\[
C_1 - C_2 = \max^\text{“expected” payoff}
\]

where \( C_1, C_2 \) are option premiums of short and long calls, respectively.

3. Actually we would pay something negative for this position because construction cost of the portfolio is negative. This means that, for the position we expect to be paid.
This is because,

\[ K_1 < K_2 \iff C_1 > C_2 \]

4. Maximum gain = \( C_1 - C_2 \)
   
   Maximum loss = \(-C_2\).

5. See the Figure in part a.

**Question 2**

1. See the diagram below:

2. The quotations are in favor of euro calls which means that market is expecting the value of euro to be higher which also makes euro calls more valuable compared to euro puts.

3. It is clear that implied volatility for euro puts is less than the implied volatility for euro calls (See the text).
4. These are the volatilities which must be plugged in (Black-Scholes) pricing formula to find the market price of options.

5. ATM volatility is the implied volatility which makes the market price of an ATM option equal to price obtained from Black-Scholes formula. Yes there are OTM vols as well. These may be different from the ATM vols, due to the presence of smiles or skews.

**Question 3**

1. See the Figure 10.20 in the text.

2. It is clear from this figure that this may be quite useful when a trader is betting on volatility movements. For example, if volatility has been unusually high for a certain period of time, and an investor may think that it may return to its “normal” level. Range binaries enable the investor to take positions on volatility. For a detailed explanation see the text.
3. This may be the case if volatility is believed to be mean reverting, and if the current volatility is higher than its mean or alternatively during the holiday seasons.

4. Unexpected spikes in volatility can cause losses.

**Question 4**

1. See the Figure 10.20 in the text for range binaries. See also Figure 10.11 for similarity between range binaries and short strangle positions.

2. When volatility is expected to be low, or when the underlying price is expected to move in a certain range.

3. For butterfly structures see Figure 10.15 in the text. It is used to hedge losses against unexpected spikes in volatility.

4. Risk arises from a short position on range binaries if volatility is high or if price moves in a very wide range.

5. This requires obtaining market data and then evaluating mark to market losses.

**Question 5**

1. They are all volatility positions. It is clear from the text that all three structures have positive payoff if underlying price moves in a certain range. Assuming that the volatility is low.

2. This is a static position (see the article in the question).

3. In the dynamic hedge, position makes money as underlying price fluctuates around the initial price. And this payoff increases as prices fluctuations are more frequent. Similarly, pay off from (long) range accrual options increases as long as price moves in a certain range.

4. By using dynamic hedging, a structure similar to range accruals can be created.
Chapter 11
Pricing Tools in Financial Engineering

Exercises

Question 1

1. If there exist positive state prices, $Q^1$, $Q^2$, $Q^3$ and $Q^4$, which satisfy the following matrix equation, then there are no arbitrage opportunities.

$$
\begin{bmatrix}
1 & 0.91 \\
0.86 & 0.77
\end{bmatrix}
\begin{bmatrix}
Q^1 \\
Q^2 \\
Q^3 \\
Q^4
\end{bmatrix}
=
\begin{bmatrix}
1.113 & 1.113 & 1.092 & 1.092 \\
0.9 & 0.92 & 0.95 & 0.96 \\
0.8 & 0.84 & 0.85 & 0.86
\end{bmatrix}
\begin{bmatrix}
Q^1 \\
Q^2 \\
Q^3 \\
Q^4
\end{bmatrix}
$$

Solving the matrix equation above, we get:

$$
\begin{bmatrix}
Q^1 \\
Q^2 \\
Q^3 \\
Q^4
\end{bmatrix}
=
\begin{bmatrix}
-0.69619 \\
0.995238 \\
1.55619 \\
-0.94524
\end{bmatrix}
$$

2. As it can easily be seen, state prices, $Q^1$ and $Q^4$ are negative which implies that there are arbitrage opportunities in this market. However, such negative state-prices can result if the chosen model is incorrect. (If for example, the wrong number of states of the wrong set of assets is chosen.)

3. 1X2 FRA rate is equal to current Libor rate of 5%. Remember that current time is $t = 1$, a 1X2 FRA starts (and expires) at time 1 and settles at time 2 and finally the current Libor rate is 5%.

Question 2

1. $\Delta = \frac{200}{5} = 40 \text{ days}$

   or

   $\Delta = \frac{40}{365} = 0.1096 \text{ years}$.  

66
By using equations (100)–(102), we can write:

\[ u = e^{\sigma \sqrt{\Delta}} \]
\[ = e^{0.18 \sqrt{0.1096}} \]
\[ = 1.061 \]

\[ d = 0.942. \]

2. 
\[ p = \frac{e^{r\Delta} - d}{u - d} = \frac{e^{0.04 \times 0.1096} - 0.942}{1.061 - 0.942} \]
\[ = 0.52 \]

3. See the tree below for the binomial tree of stock price.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
<td>80</td>
<td>120</td>
<td>160</td>
<td>200</td>
</tr>
<tr>
<td>100</td>
<td>99.95</td>
<td>99.89</td>
<td>94.2</td>
<td>94.15</td>
<td>94.1</td>
</tr>
<tr>
<td>119.44</td>
<td>119.37</td>
<td>112.57</td>
<td>112.51</td>
<td>106.1</td>
<td>106.04</td>
</tr>
<tr>
<td>126.72</td>
<td>134.45</td>
<td>112.57</td>
<td>112.51</td>
<td>106.1</td>
<td>106.04</td>
</tr>
<tr>
<td>119.44</td>
<td>119.37</td>
<td>112.57</td>
<td>112.51</td>
<td>106.1</td>
<td>106.04</td>
</tr>
</tbody>
</table>

4. Using the binomial tree for stock prices in part d, we get the following binomial tree for the call premium:
Question 3

1. In this case where the stock pays continuous dividends of 4%, there would be no changes in the binomial tree for the stock price. The tree is the same as in part (d) of the previous question. The only difference is in the calculation of up-probabilities.

\[ p = \frac{e^{(r-q)\Delta} - d}{u - d} \]

\[ = \frac{e^{(4\% - 4\%)x0.1096} - 0.942}{1.061 - 0.942} \]

\[ = 0.49 \]

where, \( q \) is the continuous rate of dividends.

By using the binomial formula to determine the call premium, we get:

\[ c = e^{-0.04(5x0.1096)} \sum_{i=0}^{5} C_i \left( \begin{array}{c} 5 \\ i \end{array} \right) p^i (1-p)^{5-i} \]
Where $c$ is the option premium at time zero and $C_i$'s are the option values on the expiration date for $i = 5, C_5 = 34.45, i = 4$ and $C_4 = 19.37$, so on.

Computing the formula given above, we obtain, $C_0 = 5.53$.

2. If stock pays 5% of its value as a dividend at the third node ($t = 120$ days), this will result in the following binomial tree for the stock price:

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 40$</th>
<th>$t = 80$</th>
<th>$t = 120$</th>
<th>$t = 160$</th>
<th>$t = 200$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>127.73</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>120.39</td>
<td></td>
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<tr>
<td></td>
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<td>113.47</td>
<td>113.40</td>
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<td>112.57</td>
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<td>106.1</td>
<td>100.74</td>
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<td>99.95</td>
<td>94.90</td>
<td>89.40</td>
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<tr>
<td>94.2</td>
<td>89.44</td>
<td>84.26</td>
<td>79.36</td>
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<tr>
<td>88.74</td>
<td>79.41</td>
<td>74.80</td>
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<tr>
<td>79.95</td>
<td>74.80</td>
<td>70.46</td>
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</tbody>
</table>

In order to compute call premium $c$ at time zero, we can apply the binomial formula in part (b) with:

$$p = 0.52$$

and

$$C_5 = 27.73, C_4 = 13.40, C_3 = 0.69$$

and

$$C_2 = C_1 = C_0 = 0.$$  

The result is:

$$c = 3.55.$$  

3. See below for the binomial tree. The third type of dividend payment creates a non-recombining tree.
Question 4

1. \( \Delta = 0.1096 \) years.

2. For this case we use equations (106)–(108), after adjusting them properly:

\[
\begin{align*}
    u &= e^{(r - rf)\Delta - \frac{1}{2} \sigma^2 \Delta + \sigma \sqrt{\Delta}} \\
    d &= e^{(r - rf)\Delta - \frac{1}{2} \sigma^2 \Delta - \sigma \sqrt{\Delta}}
\end{align*}
\]

Then,

\[
u = 1.063
\]

and

\[
d = 0.931, p = 0.5
\]

3. Using the parameters in part one and two of this question, binomial tree for exchange-rate would be:

<table>
<thead>
<tr>
<th>( t = 0 )</th>
<th>( t = 40 )</th>
<th>( t = 80 )</th>
<th>( t = 120 )</th>
<th>( t = 160 )</th>
<th>( t = 200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

4. The binomial tree for European put will be given as in the next page.

5. The binomial tree for American (put) option is somewhat more complicated than that of the European (put) option. The reason for that is that American options can be exercised earlier.
At each node, value of an American option is equal to its intrinsic value or to the value which comes from holding it until next time (node) whichever is greater.

This gives the binomial tree for the American put option, where the values are calculated backwards. The tree is shown below.

### Question 5

1. Applying equations (100)–(102), we determine $u = 1.10$, $d = 0.91$ and $p = 0.51$.

2. We can directly apply the binomial formula to determine the value of the call option. Alternatively we can use binomial tree methods. Both will lead to the same answer:

   \[ C = \$5.50 \]

3. For the barrier option, it is instructive to have a look at the binomial tree. The value of the barrier call option along the paths which lead to $\$120$ stock price becomes zero. In this case call premium will be:

   \[ C_b = \$0.041. \]
<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 40$</th>
<th>$t = 80$</th>
<th>$t = 120$</th>
<th>$t = 160$</th>
<th>$t = 200$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>max(−.50,0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>max(−.29,0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td>max(−.18,.01)</td>
<td>max(−.27,0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>max(−.08,.04)</td>
<td>max(−.06,.02)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>max(.01,.09)</td>
<td>max(.03,.07)</td>
</tr>
<tr>
<td>max(−.10,.15)</td>
<td>max(.11,.14)</td>
<td>max(.13,.08)</td>
<td>max(.20,.21)</td>
<td>max(.21,.21)</td>
<td>max(.13,.13)</td>
</tr>
<tr>
<td>max(.20,.21)</td>
<td>max(.29,.29)</td>
<td>max(.3,.21)</td>
<td>max(.37,.37)</td>
<td>max(.45,.35)</td>
<td>max(.42,.42)</td>
</tr>
</tbody>
</table>

4. Barrier option will be cheaper.
Chapter 12

Some applications of the Fundamental Theorem

Exercises

Question 1

1. In this exercise, we can easily employ the equations (66)–(70), (79)–(80) and (82)–(86). Also, remember that BDT model will yield a recombining binomial tree.

2. Using the equation,

\[ E^P[L_i] = \frac{B(0, i)}{B(0, i + 1)} - 1 \]

we get:

\[ L_0 = 5.26\% \]
\[ E^P[L_1] = 2.15\% \]
\[ E^P[L_2] = 2.20\% \]
\[ E^P[L_3] = 2.25\%. \]

Also,

\[ E^P[L_1] = 0.5x(L^u_1 + L^d_1) \]

and

\[ L^u_1 = e^{2\sigma_1} L^d_1. \]

Solving these two equations, we find

\[ L^u_1 = 2.58\% \]

and

\[ L^d_1 = 1.73\% \]

We can use the same logic to determine all possible values of the Libor rates at each node. This would lead to the following values:
At node 2:
\[
L_{uu}^2 = 3.42\%
\]
\[
L_{ud}^2 = L_{du}^2 = 2.07\%
\]
\[
L_{dd}^2 = 1.26\%
\]

At node 3:
\[
L_{uuu}^3 = 3.84\%
\]
\[
L_{uud}^3 = L_{udu}^3 = L_{duu}^3 = 2.58\%
\]
\[
L_{udd}^3 = L_{dud}^3 = L_{ddu}^3 = 1.73\%
\]
\[
L_{ddd}^3 = 1.16\%
\]

1. BDT tree for bond price of B(0,4) can be found by using the libor rates determined in the previous part (a): In the following figure, at each node, the numbers in the top box refer to money market account, and the numbers in the second box refer to price of bond at this node.

Forward price of bond which expires at time 4, at node 2 is the expected price of this bond at node 2. So forward price would be \(0.5(94.80 + 96.63) = 95.72\).
2. Price of the call option which expires at time can be calculated from
the binomial tree presented below:

![Binomial Tree Diagram]

**Question 2**

1. 

\[ L_{US} = \frac{100}{B(t, t+1)US} - 1 = 1.08\% \]

and

\[ L_{Euro} = \frac{100}{B(t, t+1)Euro} - 1 = 1.29\% \]

2. 

\[ r_t^{US} = \log(1.108) = 1.074\% \]

and

\[ r_t^{Euro} = \log(1.0129) = 1.28\%. \]
3. We need to use continuously compounded rates.


**Question 3**

1. Consider the Stochastic Differential Equation,

\[ S_{t+\Delta} = S_t + (r - r_f)S_t \Delta + \sigma S_t \Delta \sqrt{\Delta} e_t \Delta \]

and the time interval,

\[ \Delta = 1 \]

2. Assume that the following sets of random numbers are given:

\[
\begin{align*}
\{-0.4326, -1.6656, 0.1253, 0.28779, -1.1465\} \\
\{1.1909, 1.1892, -0.0376, 0.3273, 0.1746\} \\
\{-0.1867, 0.7258, -0.583, 2.1832, -0.1364\} \\
\{0.1139, 1.0668, 0.0593, -0.0956, -0.8323\} \\
\{0.2944, -1.3362, 0.7143, 1.6236, -0.6918\}.
\end{align*}
\]

These five trajectories are risk free since, i) random variables in each set have a mean of zero and ii) the equation given in part (a) has a known mean that equals interest rate differentials.

We can compute the exchange rate, \( S^j_i \), where \( j \) is the path index and \( i \) is the node (time) index. (Remember that the \( \Delta = 1 \) otherwise, the drift of the equation below will involve a \( \Delta \) as well.)

\[ S^j_i = S^j_{i-1} + (r - r_f)S^j_{i-1} + \sigma S^j_{i-1} e^j_i. \]

For example,

\[ S^1_1 = S_0(1+r-r_f+\sigma e^1_1) = 1.1015(1+0.01074-0.0128+0.15(-0.4326)) = 1.0278 \]
Similarly we can compute the remaining values for path one:
\[ S_2^1 = 0.7689, S_3^1 = 0.7818, S_4^1 = 0.8140, S_5^1 = 0.6724 \]

Path 2:
\[ S_1^2 = 1.2961, S_2^2 = 1.5247, S_3^2 = 1.5130, S_4^2 = 1.5843, S_5^2 = 1.6226 \]

Path 4:
\[ S_1^4 = 1.1181, S_2^4 = 1.2948, S_3^4 = 1.3037, S_4^4 = 1.2824, S_5^4 = 1.1198 \]

Path 5:
\[ S_1^5 = 1.1479, S_2^5 = 0.9156, S_3^5 = 1.0118, S_4^5 = 1.2562, S_5^5 = 1.1234 \]

3. Assume that we are dealing with a put option. Then, by simply checking the third number in each path, we can determine the value of the option at expiration.
Finding the expected value of the option at expiration and discounting it we get
\[ P = 0.0319 \]
(Remember that each path has a probability of 0.2.)

**Question 4**

1. By using the parameters provided in the exercise and the method employed in the previous question, we proceed as follows:

(a) We assume three time steps which implies that \( \Delta = 90 \) days.

We generated the following 5 set of random numbers:
Set 1: \{0.9501, 0.7621, 0.6154\},
Set 2: \{0.2311, \-0.4565, 0.7919\},
Set 3: \{0.6068, 0.0185, \-0.9218\},
Set 4: \{-0.4860, -0.8214, 0.7382\},
Set 5: \{0.8913, 0.4447, -0.1763\}. 

77
The values of exchange rate, on each of 5 paths, will be:

Path 1:
\[ S^1_1 = 4.1532, S^1_2 = 4.5216, S^1_3 = 4.8564 \]

Path 2:
\[ S^2_1 = 3.8835, S^2_2 = 3.7548, S^2_3 = 4.0991 \]

Path 3:
\[ S^3_1 = 4.02144, S^3_2 = 4.0822, S^3_3 = 3.7569 \]

Path 4:
\[ S^4_1 = 3.6146, S^4_2 = 3.3629, S^4_3 = 3.6532 \]

Path 5:
\[ S^5_1 = 4.1311, S^5_2 = 4.3665, S^5_3 = 4.3441 \]

Let’s assume that the option under consideration is a call option with an exercise price of 4.10 Peso/$. It is easy to see that call option expires in the money only in paths 1 and 5. So, we first compute the value of the option on each path, determine its expected value and then discount it with Mexican interest rate. Hence, the call premium is 0.1884.

2. This information is important in a sense that it is an additional risk factor which affects the exchange rate process.

3. Let’s first have look at the distribution of the reserves. Let \( R_t \) be the level of reserves. Then the distribution of \( \log(R_t) \) is

\[ \phi [\log(R_0) + (\mu - \frac{\sigma^2}{2})T, \sigma \sqrt{T}] \]

So probability that \( R_t < 6 \) (probability of experiencing a one shot devaluation) is less than 2%. Even though the information is important in pricing the option, it is very hard to determine its effect when we use only 5 paths with only 3 steps.

So we need to increase the number of paths and steps to allow for such an event with very low probability. We can proceed in two ways. Either
we can use the same random variables generated for the exchange rate process to compute the reserves.

Here we assume that there is a perfect correlation between two processes. Or, alternatively, we can generate a separate set of random variables which implies that there is no correlation between the two.
Chapter 13

Fixed Income Engineering

Exercises

Question 1

1. In order to compute the bond prices from FRA rates, we can simply use the equation (23) in the chapter.

\[ B(t_0, t_i) = \frac{100}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j))} \]

where

\[ \delta = 0.25 \text{ years} \]

and \( F(t_0, t_i) \) is the \( i \times (i + 1) \) FRA rate.

Using the equation we will get the following discount bond prices:

<table>
<thead>
<tr>
<th>Term</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>99.01</td>
<td>99.01</td>
</tr>
<tr>
<td>6</td>
<td>97.88</td>
<td>97.91</td>
</tr>
<tr>
<td>9</td>
<td>96.72</td>
<td>96.77</td>
</tr>
<tr>
<td>12</td>
<td>95.51</td>
<td>95.58</td>
</tr>
<tr>
<td>15</td>
<td>94.16</td>
<td>94.28</td>
</tr>
<tr>
<td>18</td>
<td>92.70</td>
<td>92.86</td>
</tr>
</tbody>
</table>

2. We use the following relationship between discount bond prices and the yield:

\[ B(t_0, t_i) = \frac{100}{1 + \delta_i r_i} \]

where \( \delta_i \) is the time to maturity as a fraction of a year, and \( r_i \) is the corresponding interest rate (yield) for that maturity.
Rewriting the equation in terms of $r_i$, we get

\[ r_i = \frac{1}{\delta_i} \left( \frac{100}{B(t_0, t_i)} - 1 \right) \]

Using this equation we obtain the yields shown in the first Table below.

<table>
<thead>
<tr>
<th>Term</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>6</td>
<td>4.27</td>
<td>4.32</td>
</tr>
<tr>
<td>9</td>
<td>4.45</td>
<td>4.52</td>
</tr>
<tr>
<td>12</td>
<td>4.63</td>
<td>4.71</td>
</tr>
<tr>
<td>15</td>
<td>4.85</td>
<td>4.96</td>
</tr>
<tr>
<td>18</td>
<td>5.12</td>
<td>5.25</td>
</tr>
</tbody>
</table>

3. Computation of swap rates is explicitly shown in the text. We can apply equations (25)–(31). This would yield the swap curve which is shown in the second Table below.

<table>
<thead>
<tr>
<th>Term</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4.50</td>
<td>4.60</td>
</tr>
<tr>
<td>9</td>
<td>4.60</td>
<td>4.70</td>
</tr>
<tr>
<td>12</td>
<td>4.73</td>
<td>4.83</td>
</tr>
<tr>
<td>15</td>
<td>4.92</td>
<td>5.04</td>
</tr>
<tr>
<td>18</td>
<td>5.15</td>
<td>5.29</td>
</tr>
</tbody>
</table>

4. We can easily see that curves computed in part (1) and (3) are not the same. The following graph shows this:

5. See appendix for the calculation of the par yield curve. We can derive

\[ y_n = \frac{1}{\delta} \left( \frac{100 - B(0, n)}{\sum_{i=1}^{n} B(0, i)} \right) \]
Yield curve vs Swap curve (Bid)

<table>
<thead>
<tr>
<th>Term</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>6</td>
<td>4.25</td>
<td>4.30</td>
</tr>
<tr>
<td>9</td>
<td>4.40</td>
<td>4.46</td>
</tr>
<tr>
<td>12</td>
<td>4.55</td>
<td>4.62</td>
</tr>
<tr>
<td>15</td>
<td>4.73</td>
<td>4.83</td>
</tr>
<tr>
<td>18</td>
<td>4.95</td>
<td>5.07</td>
</tr>
</tbody>
</table>
from the equation provided in the appendix ($\delta = 0.25$ years).

This leads to:

6. The computation of zero-coupon yield curve is very similar to computation of yield curve from discount bond except that the yield this time corresponds to a formula that uses Bond-equivalent yields:

$$B(t_0, t_i) = \frac{100}{(1 + y_i)^{\delta_i}}$$

where $\delta_i$ is the same as in part (2). Solving this equation for $y_i$, we obtain:

$$y_i = \left( \left( \frac{100}{B(t_0, t_i)} \right) \frac{1}{\delta_i} - 1 \right).$$

By using the formula just derived, the zero coupon rates are computed as in the Table below.

(You may, alternatively, follow the method described in the Appendix to calculate the zero-coupon yield curve from par yield curve).

<table>
<thead>
<tr>
<th>Zero-Coupon Yield Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Term</strong></td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>18</td>
</tr>
</tbody>
</table>

**Question 2**

1. Let’s start by considering the following two portfolios:
   Portfolio A: Nominal = $100. Long position in $(0 \times 1)$ and $(1 \times 2)$ FRA’s.
   Portfolio B: Nominal = $100. Long position in plain vanilla swap.
Portfolio A:
Since we start at time zero, we already know the current Libor rate $L_0$ and $(0 \times 1)$ FRA rate. They are both equal to 5%. But, we do not know $L_1$ and the $(1 \times 2)$ FRA rate.

\[
\begin{array}{c|c}
L_0 & L_1 \\
5\% & ? \\
\hline
5\% & \text{F}_{1\times2}
\end{array}
\]

Portfolio B:

\[
\begin{array}{c|c}
L_0 & L_1 \\
5\% & ? \\
\hline
5\% & \text{F}_{1\times2}
\end{array}
\]

From the text we know that both structures are very similar to each other. We can use that similarity to derive the $(1 \times 2)$ FRA rate. Indeed, we have the following relationship between FRA and swap rate: (See equation (17) and explain why we can’t use it)
$100 \left( \frac{5\%}{1.05} + \frac{F_{1x2}}{(1.05)(1 + F_{1x2})} \right) = $100 \left( \frac{6.2\%}{1.05} + \frac{6.2\%}{(1.05)(1 + F_{1x2})} \right)

Simplifying and rearranging:

\[ F_{1x2} = \frac{6.2\% + (6.2\% - 5\%)}{1 - (6.2\% - 5\%)} = 7.49\%

In order to write a general formula, it would be helpful to write the same equation for \( F_{2x3} \).

\[ F_{2x3} = \frac{6.4\% + (6.4\% - 5\%)x(1.0749) + (6.4\% - 7.49\%)}{1 - ((6.4\% - 5\%)x(1.0749) + (6.4\% - 7.49\%))} = 6.84\%

Then a general formula would be in the form of:

\[ F_{(n-1)x(n)} = \frac{S_n + \sum_{i=1}^{n-1} [(S_n - F_{(i-1)x_i}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)x_j})]}{1 - \sum_{i=1}^{n-1} [(S_n - F_{(i-1)x_i}) \prod_{j=i+1}^{n-1} (1 + F_{(j-1)x_j})]}

Using the general formula just derived, we obtain the following FRA rates (We used bid quotations for the corresponding swap rate):

<table>
<thead>
<tr>
<th>Term</th>
<th>FRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0X1</td>
<td>5.00</td>
</tr>
<tr>
<td>1X2</td>
<td>7.49</td>
</tr>
<tr>
<td>2X3</td>
<td>6.84</td>
</tr>
<tr>
<td>3X4</td>
<td>9.11</td>
</tr>
<tr>
<td>4X5</td>
<td>9.98</td>
</tr>
<tr>
<td>5X6</td>
<td>12.11</td>
</tr>
</tbody>
</table>

After computing the FRA rates from swap curve, the rest is relatively easy to handle.
2. We can follow the same method used in the previous question part (1). As a result we get the answers shown in the Table below.

3. See question one, part (2) for the details:

<table>
<thead>
<tr>
<th>Term</th>
<th>Bond Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>95.24</td>
</tr>
<tr>
<td>2</td>
<td>88.60</td>
</tr>
<tr>
<td>3</td>
<td>82.93</td>
</tr>
<tr>
<td>4</td>
<td>76.01</td>
</tr>
<tr>
<td>5</td>
<td>69.11</td>
</tr>
<tr>
<td>6</td>
<td>61.64</td>
</tr>
</tbody>
</table>

4. See the previous question, part (5):

<table>
<thead>
<tr>
<th>Term</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00</td>
</tr>
<tr>
<td>2</td>
<td>6.43</td>
</tr>
<tr>
<td>3</td>
<td>6.86</td>
</tr>
<tr>
<td>4</td>
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<tr>
<td>5</td>
<td>8.94</td>
</tr>
<tr>
<td>6</td>
<td>10.37</td>
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</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>Par Yield Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00</td>
</tr>
<tr>
<td>2</td>
<td>6.20</td>
</tr>
<tr>
<td>3</td>
<td>6.40</td>
</tr>
<tr>
<td>4</td>
<td>7.00</td>
</tr>
<tr>
<td>5</td>
<td>7.50</td>
</tr>
<tr>
<td>6</td>
<td>8.10</td>
</tr>
</tbody>
</table>
**Question 3**

1. The bid-ask spread on a forward start swap is equal to $S^{ask} - S^{bid}$. We can use the equations (25)–(31) to compute $S^{ask}$ and $S^{bid}$.

$$S^{bid}_{2 \times 5} = \frac{B(0, 3) \times F_{2 \times 3} + B(0, 4)F_{3 \times 4} + B(0, 5)F_{4 \times 5}}{B(0, 3) + B(0, 4) + B(0, 5)}$$

$$= \frac{82.93 \times 6.84\% + 76.01 \times 9.11\% + 69.11 \times 9.98\%}{82.93 + 76.01 + 69.11}$$

$$= 8.55\%$$

The $S^{ask}$ can be computed in a similar way.

2. The price of the coupon bond at time zero is given by

$$P(0, 4) = \left( \sum_{i=1}^{4} B(0, i) \times \$7 \right) + B(0, 4) \times \$100$$

$$= \$100.$$

A trader who wants to (forward) sell this coupon bond at a price of $F$, to be delivered at time 2, may want to hedge himself. So this trader can buy this coupon bond in the spot market today and “store” it. Since its price is $100, he needs to borrow $100 for funding purposes.

When the trader receives coupon payments at time 1 and 2 and the forward price amount, $F$ of the coupon bond, the trader will repay the initial $100. This means that the trader needs to borrow:

$$B(0, 1) \times \$7 + B(0, 2) \times (\$7 + F) = \$100.$$

Solving this equation we find,

$$F = \$98.34$$
Chapter 14
TOOLS FOR VOLATILITY ENGINEERING, VOLATILITY SWAPS, AND VOLATILITY TRADING

EXERCISES

Question 1

1. Let’s look at the cash flow of the volatility (variance) spread swap:

\[ -(\sigma_{\text{Nasdaq}}^2 - \sigma_{\text{S&P500}}^2)N2 \]

It is clear from this expression that investor actually takes a long position on the S&P500 variance and a short position on the NASDAQ variance. This trade can be put in place by simultaneously entering into a long S&P500 variance swap and a short NASDAQ variance swap.

Pricing here is to determine the initial variance swap spread which makes the initial value of the swap between the two indices have a value of zero. This price is given as 21% in the question.

2. Yes, we need the correlation between the two markets.

If the correlation is high (close to one in absolute value) between these markets, then this implies that most of the time, volatility will move in both markets in the same direction which in return, indicates that volatility (variance) spread is relatively tight in the long run (This makes the position mentioned in the question reasonable). So the fixed leg of the spread has to be set (relatively) higher.

If the correlation is low (close to zero in absolute value), which implies that these two markets move more or less independently from each other, then there is no reason to believe that the volatility spread between the markets should get narrower.

It is less likely that the investor who holds a long position will end up with a positive payoff. In order to make the initial value of the swap equal to zero, the fixed leg of the spread needs to be set at a lower level.

3. The smile effect is important. However, in this present case the trade concerns realized volatility and not the Black-Scholes implied volatility. This means that the pricing of the swap will make no use of the smile in any direct way. Indirectly, the smile can be useful to calibrate a model, on the other hand.
4. If the position is taken by using volatility (variance) swaps, then this may be less risky compared to the other ways of taking the same position. Also the pricing of the instrument may be easier. (See the exercise for alternative ways of taking the same position and see the text for the risk of these positions).

The main risk involved here is related to the assumption that the long-run dynamics of the volatility spread is stationary. If this underlying assumption is violated, then the position may lose.

**Question 2**

1. If an investor buys long-dated volatility, and sells short-dated volatility, then the investor is expecting a decrease in the short dated volatility and an increase in the long dated volatility.

   Of course, there is no guarantee that these expectations will be realized. If short run volatility goes up or and long run volatility decreases, the pay off from the position would definitely be negative.

2. This is equivalent to an investor constructing a short straddle position by using knock-out options. Long straddle may be constructed by using options which have break-out clause to put a limit to unrestricted risk of loss that arises from the short (straddle) position in the short run.

   If short dated volatility turns out to be high, an additional premium can be triggered on the options which are used for the long straddle. This additional payoff from the long volatility position off sets the losses from the short volatility position.

3. The relevant payoff function will shift upwards by the amount of the additional payoff.

4. If the additional premium is a fixed amount, this may cut potential losses, yet it may not be sufficient enough. However, considering that the short position is taken for one month, this risk may not be a very big risk.

5. It is clear from the text (see section 3.3) that volatility positions taken by using the straddles are not pure volatility positions.
Chapter 15

Smile Effects in Financial Engineering

Exercises

Question 1

1. In this question the implied volatilities are computed by using a risk-free interest rate of 2%. The calculations are summarized by the following figure.

![Volatility smile put-bid/ask](image)

2. The calculations yield the following results in the Figure shown below. The bid-ask spread on the implied volatilities can be seen in the following Figure. Clearly, these spreads are not constant across options with different moneyness.
3. When the two figures are put together the result will be more like a skew.
Question 2

1. Cap/floor volatility is thought to be higher than swaption volatility because, the market buys volatility trough swaptions and sells volatility trough cap/floors. Everything else being the same the bid-ask difference should make cap/floor volatility a little higher.

(a) A callable bond has a higher coupon because the bond incorporates a short swaption. If rates fall below a level, the issuer has the right to call the bond at par. From bondholder’s point of view this is equivalent to selling a swaption. The option holder has the right to get into a fixed receiver swap that pays the same coupon at the call date.

This embedded option will have a premium and this premium will make the coupon of the callable bond higher. Investors may consider these higher coupons as yield enhancement and buy these bonds. (Callable bonds are usually not callable for a certain period after the issue date. During this period the investor will receive the high coupon regardless of the interest rate movements.)
(b) On the other hand caps/floors are used by corporates to hedge the interest rate risk. So, the market will structurally be short caps/floors but long swaptions.

2. Market sells financial services to corporates. One of these services is helping them hedge interest rate risk. This is done using caps and floors.

On the other hand, market sells yield enhancement products to investors. These structures are often callable, which results in the embedded swaption.

3. This statement is an example of hedging and risk managing of volatility risk. The industry is buying and selling volatilities and meeting different needs of its clients. However, the two risks are not identical and the “volatility books” need to be carefully managed in order to be properly hedged.

Also, note that from the point of view of a volatility trader the same risk can be hedged in (many) different ways and by using different financial instruments. Some alternatives can be cheaper than others.
Chapter 16
HOW DO CREDIT DERIVATIVES CHANGE FINANCIAL ENGINEERING?

EXERCISES

Question 1
1. The standard approach here is to calculate some conventional ratios. These ratios can then be used along with regression analysis to estimate the default probability.

2. To obtain the migration matrix for a particular credit rating, once could look at the past ratings data on all corporates and calculate the statistical estimates for the transition probabilities from one rating to another in a given time period.

3. There are many credit ratings and each credit comes with a migration matrix. Using past data one can also calculate the joint probabilities that two or more credits migrate to a different rating.

Question 2
1. See figure 5.6 and the related example in section 3.2.1 in the text.

2. See figure 5.5 and the related example in section 3.2 in the text.

3. Yes, a swaption will be needed. The main reason being that Bond A is callable after 3 years and matures in 4 years whereas Bond B matures in 5 years. It is clear that if interest rates decrease substantially, Bond A will be called.

4. First, let’s look at the required structure to convert Bond A in to Bond B:

(a) To eliminate the credit risk involved in Bond A, we need to buy a CDS with 4 year maturity. That will change the credit of the initial portfolio from AA- to AAA.
(b) Second step involves the conversion of fixed rate in to floating rate. For this we need a (fixed payer) interest rate swap in USD with maturity 5 years.

(c) By using a cross currency swap (floating to floating), we exchange USD floating into DEM floating. So we need a currency swap for 5 years.

(d) Finally, we need to hedge the risk mentioned in part (c). So we should buy a (Bermudan type) swaption in USD.

Question 3

(a) A typical cash flow diagram will incorporate the following. If you are short the CDO, then you receive a fixed amount at the initial point \( t_0 \). Then you make payments made of a floating risk-free rate plus a fixed spread. However, if one or more of the underlying credits default your share of the defaulted amount will be deducted from the coupon.

(b) Since you are receiving a spread over a floating rate, the interest rate risk is minimal. There is some risk only between the coupon payments dates. This can be hedged using strips of FRA’s. Or, by using swaps as the reading suggests. Otherwise the CDO is an investment vehicle and the investor is exposed to changes in the credit curve. If needed, such risks can be hedged by taking positions on a proper set of CDSs.

(c) A decline of the overall level of interest rates means the floating rates are going down. If the investor is hedged through the FRA’s this will have no effect on the overall returns. On the other hand if default rates increase the value of the CDO will decline.

(d) As underlying credits default this will decrease the principal amount involved in the CDO during its life cycle. On the other hand if such a CDO is hedged using a swap, the swap notional will remain fixed.
This means that a plain vanilla swap will end up introducing a basis risk. However, a customized swap where the swap notional decreases as CDO principal changes will be more expensive.
Chapter 16

Credit Markets: CDS Engineering


4. (a) i. The following figure shows the cash flows involved in the trade.

ii. Clearly a negative basis trade is going on, leading to a risk-free gain. ING claims that while CDS rate $c_{t_0}$ has decreased by 5bps, Attentat 2010 bond price has increased 1bp recently which is an anomaly. So, according to ING the CDS rate should eventually increase leading to gains from the current trade.

iii. The bond is bought for 100 which pays fixed coupons $C_{t_0}$ at each time point if it doesn’t default. But it is default protected through the CDS by a payment of $midas + 27bp$. The expected return is $C_{t_0} - c_{t_0}$ at each coupon payment date. (Assuming that the insurer does not default.)

iv. There might have been a surge of structured produce like CDOs being issued in the market which narrowed the CDS spread where as the bond spread has actually increased.
(b) The LBBW is grandfathered and hence is guaranteed to provide the said returns on the holding. So if a trade neutral position is held by being short in DG Hyp 4.25% and long in LBBW 3.5% (under swaps), as spreads tightens, the gains from LBBW will remain the same but the payments for being short in DG Hyp will decline leading to eventual gains from holding this position.

In the second case Land Berlin 2.75% is risk protected and hence the same logic as the previous case holds.

(c) Note that the trade has an yearly positive carry. Besides the 5-year iTraxx spread 38bp is considered to be high now, considering the forward rates. Hence they are expected to drop in the future. Thus this trade results in an advantage for the investor if implied forward rates are higher than they should be. Observe the graph below.
(a) Credit linked notes are assets issued by financial institutions which have exposure to the credit risk of a reference Issuer. These notes pay an enhanced rate to the investor for taking on this additional credit risk. If the Reference Issuer defaults then the investor receive the recovery price of the reference security. Most credit linked notes are issued as traditional medium term notes that contain embedded credit default swaps.

**Example:**

*Drug company Credit Linked Note*

**Termination Date:** 5 Years

**Reference Security:** ABC bond 7.125% maturing 12/1/09

**Interest:** Libor + 50bp per annum. Interest paid quarterly.

*In this CLN the investor receives Libor + 50bp coupon against the exposure to the default risk of the drug company in an asset backed trust.*

Thus a CLN will be preferable to a straight Treasury if the investor desires a higher coupon associated with the risk of default of a certain security.

(b) Investment banks may need to hedge their position during the issuance process as the readings suggest. The process of packaging, pricing and selling a CLN may take time and during this period markets may move.

As dealers issue CLNs they will have short derivatives positions mentioned in the text, in particular they will have short CDs positions.

(c) In the text the climate has been influenced by a shortage of corporate bonds in the secondary market.

Note that when this happens the value of these bonds would go up and the associated “risk premium” would decline. As discussed in Chapter 16, this would lead to a lowering of the associated CDS rate.
(d) The arbitrage mentioned in this reading is in fact a true arbitrage in the academic sense. The logic of the arbitrage argument goes as follows.

A basket of corporate bonds is something similar to a synthetic CLN packaged using the same names by a dealer. They both have exposure to an identical set of credits, they contain very similar default risks and they pay an enhanced coupon. Hence in theory the difference between the coupon paid by the basket and the synthetic CLN should be the same.

Yet, as discussed in the chapter technical factors may make the two coupon diverge from each other. This is called the bond basis. The text mentions that this basis was negative originally and became even more negative as a result of the activity mentioned in the reading.

Those dealers who had access to the underlying bonds could then put together packages, by selling and buying the underlying and the associated CLN.

To represent the cash flows in a graphic one would use Figure 16-5 and put it together with the cash flows of a risky bond, which will be similar.

(e) If some bonds become special in the repo market, this means that there is a great deal of demand for them. Under such conditions those players who want to borrow these bonds will be willing to “surrender” their cash to the repo dealer at zero interest.

(f) Because if they have the bonds in their portfolio they give to bond to repo dealer and receive cash at zero cost, instead of paying interest to banks.
Chapter 17

ESSENTIALS OF STRUCTURED PRODUCT ENGINEERING

1. (a) Let $r_m$ denote the $m$ month swap rate (or Libor rate). Then the $3 \times n$ month forward rate $f_{(3 \times n)}$ is calculated thus:

$$(1 + r_3 \frac{3}{12})(1 + f_{(3 \times n)} \frac{n-3}{12}) = (1 + r_n \times \frac{n}{12}).$$

The discount curve is calculated thus: the $n$- month discount rate $B(t_0, t_n)$ is

$$B(t_0, t_n) = \frac{1}{\prod_{i=0}^{n-1} (1 + r_i/12)}.$$

(b) The $24 \times n$ month forward rate $f_{(24 \times n)}$ is calculated thus:

$$(1 + r_3 \frac{24}{12})(1 + f_{(24 \times n)} \frac{n-24}{12}) = (1 + r_n \times \frac{n}{12}).$$

(c) The components for this note is a discount curve, a 2-year forward curve, a market for CMS swaps and Bermuda swaptions (since the note is callable).

(d) Let $cms_{j,k}^i$ denote $j$-year CMS bought for $k$ years evaluated at the $i$-th year. Let $c_{t_0}$ be the premium for a 2-year Bermuda swaption. Calculate $R_1 = L_0 + \alpha_0$, where $L_0$ is the current 1-year Libor rate and $\alpha_0$ is calculated as:

$$c_{t_0} = \alpha_0 + B(t_0, t_1)\alpha_0 + B(t_0, t_2)\alpha_0.$$

Year 2 coupon is: $\alpha_1(cms_{2,3}^2) + L_0 + \alpha_0$. We know $cms_{2,3}^2$ and hence $\alpha_1$ is calculated by equating:

$$\alpha_1(cms_{2,3}^2) + L_0 = cms_{2,3}^2$$

giving $\alpha_1 = 1 - \frac{L_0}{cms_{2,3}^2}$. Similarly $\alpha_2$ is calculated from

$$\alpha_1(cms_{2,3}^2) + \alpha_2(cms_{3,3}^3) + L_0 = cms_{3,3}^3$$

giving $\alpha_2 = 1 - \frac{cms_{2,3}^2}{cms_{3,3}^3}$. 

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(e) The components for this note is a discount curve, a 3-year and a 2-year forward curve, a market for CMS swaps and Bermuda swaptions (since the note is callable).

(f) Let $c_{t_0}$ be the premium for a 2-year Bermuda swaption. Calculate \( R_1 = L_0 + \alpha_0 \), where \( L_0 \) is the current 1-year Libor rate and \( \alpha_0 \) is calculated as:

\[
c_{t_0} = \alpha_0 + B(t_0, t_1)\beta_1 + B(t_0, t_2)\beta_2.
\]

Here \( c_{t_0}, B(t_0, t_1), B(t_0, t_2) \) are known and the rest \( \alpha_0, \beta_1, \beta_2 \) has to be determined from that. \( \alpha \) is calculated as \( \alpha = \frac{\text{cms}^{3,3} - \text{cms}^{2,3}}{s_0^3} \)

where \( s_0^3 \) is the 3-year swap rate.

(g) \( \beta_1 = \beta_2 \) can be chosen appropriately to satisfy \( c_{t_0} = \alpha_0 + B(t_0, t_1)\beta_1 + B(t_0, t_2)\beta_2 \).

2. (a) The note can be engineered thus:
   - 1-year Libor deposit.
   - Get into a receiver interest rate swap paying Libor and getting 5.23%.
   - Buy digital cap (for Libor > 6.13%) and digital floor (for Libor > 6.13%).
   - Pay CMS10 for first 2 years and 8 × CMS10 for the next three years.
   - Receive CMS30 for first 2 years and 8 × CMS30 for the next three years.

(b) An investor who expects the yield curve to steepen in the long run and expects Libor to remain quite high would demand this product. She/he would expect Libor to increase and also the CMS spread to increase leading to an increase in the difference \( CMS30 - CMS10 \).

3. (a) The investor expects that the CMS10 rate would gradually increase.

(b) If the CMS10 curve flattens or the rate does not increase significantly (slow increase) then the coupons will go on decreasing and might become negligible.
The other risk is that the issuer might not call the note in such a situation and the investment would be stuck having sub-optimal gains.

(c) (typo in problem, should be CMS10 instead of Libor, and we have an additional rate 10%) The following table gives the coupons, given that we know the CMS rates as given in the problem:

<table>
<thead>
<tr>
<th>Year</th>
<th>Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>9.00%</td>
</tr>
<tr>
<td>Year 1, Q2</td>
<td>(9.00 + 5.00 - 4.65)% = 9.35%</td>
</tr>
<tr>
<td>Year 1, Q3</td>
<td>(9.35 + 6.00 - 4.85)% = 10.50%</td>
</tr>
<tr>
<td>Year 1, Q4</td>
<td>(10.50 + 6.50 - 5.25)% = 11.75%</td>
</tr>
<tr>
<td>Year 2, Q1</td>
<td>(11.75 + 7.00 - 5.45)% = 13.30%</td>
</tr>
<tr>
<td>Year 2, Q2</td>
<td>(13.30 + 8.00 - 5.65)% = 15.65%</td>
</tr>
<tr>
<td>Year 2, Q3</td>
<td>(15.65 + 9.00 - 5.65)% = 19.00%</td>
</tr>
<tr>
<td>Year 2, Q4</td>
<td>(19.00 + 10.00 - 5.65)% = 23.35%</td>
</tr>
<tr>
<td>Year 3</td>
<td>(23.35 + 10.00 - 5.65)% = 27.70%</td>
</tr>
<tr>
<td>Year 4 - 10</td>
<td>(23.35 + 10.00 - 5.65)% = 27.70%</td>
</tr>
</tbody>
</table>

So, as the CMS10 rate increases the coupon payment snowballs. The following table gives the coupons, given that we know the CMS rates are constant at 3.5% as given in the problem:

<table>
<thead>
<tr>
<th>Year</th>
<th>Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>9.00%</td>
</tr>
<tr>
<td>Year 1, Q2</td>
<td>(9.00 + 3.50 - 4.65)% = 7.85%</td>
</tr>
<tr>
<td>Year 1, Q3</td>
<td>(7.85 + 3.50 - 4.85)% = 6.50%</td>
</tr>
<tr>
<td>Year 1, Q4</td>
<td>(6.50 + 3.50 - 5.25)% = 4.85%</td>
</tr>
<tr>
<td>Year 2, Q1</td>
<td>(4.85 + 3.50 - 5.45)% = 2.90%</td>
</tr>
<tr>
<td>Year 2, Q2</td>
<td>(2.90 + 3.50 - 5.65)% = 0.75%</td>
</tr>
<tr>
<td>Year 2, Q3</td>
<td>max(0.75 + 3.50 - 5.65, 0)% = 0%</td>
</tr>
<tr>
<td>Year 2, Q4</td>
<td>0%</td>
</tr>
<tr>
<td>Year 3</td>
<td>0%</td>
</tr>
<tr>
<td>Year 4 - 10</td>
<td>0%</td>
</tr>
</tbody>
</table>

Since the CMS10 does not increase, the coupon value decreases and reaches 0%.

(d) This coupon can be characterized by an interest rate swap. Note that the CMS10 rates are all known. Hence the coupons are all fixed in the beginning. The investor can pay the fixed coupons and receive Libor at every coupon payment date.
(e) If the 8 FRAs are known, we can calculate the discount curve from that. Now if the coupons at each time $t_i$ is $c_{t_i}$, then the price of the coupon at $t_i$ is

$$P_i = B(t_0, t_i)(c_{t_i} - F_{t_i})$$

where $F_{t_i}$ is the forward rate at $t_i$.

(f) The first years coupon is generated from Libor and a part of the price of the Bermuda swaption sold.

(g) The coupons are floored as mentioned (“minimum of 0%”)

(h) CONTRACTUAL EQUATION:

**Snowball Note = Libor deposit + Receiver interest rate swap (wrt CMS10) + Short Bermuda swaption**

4. The *CMS spread note* can be engineered thus:

- Deposit $10 million earning Libor.
- Get into a 10 year Receiver swap.
- Receive $16 \times CMS30$ and Pay $16 \times CMS10$ for 10 years with two CMS swaps.
- Sell a Bermuda swaption.
- Buy a CMS spread floor and a CMS cap.

(a) The investor expects the yield curve will steepen in the long run.

(b) The obvious risk is if the yield curve flattens then there will be very low yielding coupons. Also the coupon may not be called while a low yield is going on. Of less significance but still to be remembered is that in case of very steep yield curve, the gains will still be capped at 30%.

5. (a) This coupon can be engineered with the help of knock-in and knock-out put options.

(b) This product does depend on forward volatility since the pricing and hedging of these options are dependent on volatility.
Chapter 18
CREDIT INDICES AND THEIR TRANCHES

1. (a) A barbell is a strategy of maintaining a portfolio of securities concentrated at two extremes in terms of maturity date: very short-term and very long term.
   A positive roll down is a positive return from a security trading at a discount which reaches its par value near the maturity date. Time decay is the ratio of the price with respect to a decrease in time to expiration of any asset whose value decreases over time.

   (b) A jump to default occurs when an investment grade entity with high rating which has been continuing in subsequent rolls of a credit index without degradation (in credit rating) suddenly defaults.

   (c) SG’s strategy: Since the 7-year equity correlation has tightened, the spread has increased against the 5-year and 10-year spreads. Hence it is profitable to sell the 7-year equity tranche protection and buy the 5-year and 10-year equity tranche protection barbell. Naturally there is a steepening of the 7-year spread. A jump to default is well protected by the 10-year protection.
   SG thinks that Alstom’s 3-5 year curve is steep. This means Alstom March 2010 bonds at 6.25% would lead to mark-to-market gains. The 3-year CDS provides the protection against default.

2. (a) Suppose we have a portfolio of \( n \) names with some default correlation \( \rho \). The risk of the entire portfolio moves according to the change in default correlation. On the other hand if the portfolio is tranched according to the order of their defaults then the various tranches behave differently as \( \rho \) changes. For example, a high \( \rho \) indicates subsequent defaults occurring together where as a low \( \rho \) makes occurrence of subsequent defaults more or less independent.

   (b) As default correlation increases the area under the middle part of the default density function decreases and the mass on the two extreme tranches increase. Hence area under the subordinate
tranche increases, which means that the probability that the subordinate tranche loses all its money decreases. Hence the risk for this tranche decreases along with the spread.

On the other hand the cushion for the senior tranches decrease as default correlation increases. Hence it is more likely that the the protection seller for the senior tranches will concede some losses. Hence the risk and hence the spread for this tranche increases.

(c) With a decrease in default correlation one should long on protection on the subordinate tranches and short on protection on the senior tranches.

3. (a) iTraxx is a group of credit derivative indices managed by the International Index Company (IIC) and covering Europe, Asia and Australia. The entities in the portfolio forming the indices are selected on the basis of trading volume and liquidity of the underlying CDS. For example the iTraxx Europe index comprises of 125 investment grade names.

(b) A standard tranche for a credit index is a tranche with pre-specified lower and upper attachment point making it much more liquid than tranches created individually by negotiating with market makers. The following are the standard attachment points signifying the percentage of defaults protected by the the seller of the tranche:

- **Equity tranche**: first $0 - 3\%$
- **Mezzanine tranche**: $3 - 6\%$
- **Senior tranche**: $6 - 9\%$
- **Super senior tranche**: $9 - 12\%$

(c) We can mention the following few differences:

- Standardized tranches of credit indices are unfunded and hence no cash payment is involved whereas tranches of CDO’s issued in the market by banks or hedge funds may be funded and require cash payment.

- For the standardized tranches the percentage of default that it protects is already determined, on the other hand tranches of CDO’s issued in the marketplace depends on the issuer (bank or hedge fund).
• The underlying portfolios are likely to be different.
• The standard tranches are more liquid than tranches issued by banks and hedge funds.
Chapter 19

Defaults Correlation Pricing and Trading

1. The 1-year CDS rates in terms of basis points for the three IG names in the portfolio are given by:

\[ c(1) = 15, c(2) = 11, c(3) = 330. \]

The recovery rate is uniformly 40%, i.e., \( R = 0.4 \). In every tranche a notional amount of $1.50 is invested.

(a) The default probabilities are given by:

\[ p_1 = \frac{c(1)/10000}{1 - R} = \frac{0.0015}{0.6} = 0.0025, \]
\[ p_2 = \frac{c(2)/10000}{1 - R} = \frac{0.0011}{0.6} = 0.001833, \]
\[ p_3 = \frac{c(3)/10000}{1 - R} = \frac{0.033}{0.6} = 0.055. \]

(b) To obtain the default distribution we need to get the default correlations between the three names first. If the correlations are all zero or all one (not possible in this case as the default probabilities are different) we can calculate the default distribution using standard probabilistic techniques. If they are different from all zero or all one, then we can use the latent variable technique described in this chapter to generate dependent zero-one-valued random variables and simulate the default distribution from that.

(c) The defaults are uncorrelated. Let \( D \) denote the number of defaults. Then

\[
\begin{align*}
P(D = 0) &= (1 - p_1)(1 - p_2)(1 - p_3) = 0.9409093, \\
P(D = 1) &= p_1(1 - p_2)(1 - p_3) + (1 - p_1)p_2(1 - p_3) \\
&\quad + (1 - p_1)(1 - p_2) * p_3 \\
&= 0.05884826, \\
P(D = 2) &= p_1p_2(1 - p_3) + (1 - p_1)p_2p_3 + p_1(1 - p_2) * p_3 \\
&= 0.00024216, \\
P(D = 3) &= p_1p_2p_3 = 0.000000252.
\end{align*}
\]
(d) A $0 - 66\%$ tranche provides protection on the first two defaults. The expected loss (in dollars) on this tranche is given by:

\[
L_{0-66} = 0 \times P(D = 0) + (\frac{1}{2} \times P(D = 1) + 1 \times P(D \geq 2)) \times 1.50 \times (1 - R) = 0.02669989.
\]

(e) The pay over (in dollars) for the $0 - 50\%$ and $50 - 100\%$ tranches are:

\[
L_{0-50} = 0 \times P(D = 0) + 1.50 \times P(D \geq 1) \times (1 - R) = 0.05318163,
\]

\[
L_{50-100} = 0 \times P(D \leq 1) + \left(\frac{2}{3} \times P(D = 2) + 1 \times P(D = 3)\right) \times 1.50 \times (1 - R) = 0.0001455.
\]

(f) We have \(c(1) = c(2) = c(3) = 100\). Hence \(p = p_1 = p_2 = p_3 = \frac{100}{10000} = 0.01667\). The default distribution is given by:

\[
P(D = 0) = (1 - p) = 0.98333,
\]

\[
P(D = 3) = 0.01667.
\]

There is only one tranche in this case which has spread:

\[P(D = 3) \times 0.6 = 0.01,\] i.e., 100bp, whose expected loss is \$1.50 \times P(D = 3) \times 0.6 = \$0.015.

2. Let the principal amount to be invested (in dollars) be \(N = 100\). Recall that the iTraxx standard equity tranche is quoted in terms of upfront percentage of the notional amount. Hence \(q_{t_0} = 14\%, q_{t_1} = 14\%, q_{t_2} = 5\%, q_{t_3} = 16\%\). Assume the 5-year zero coupon Treasury rate is \(r_t = 5\%\), which will be the risk-free bond for our purpose. The Libor rate is fixed at \(L_t = 5\%\). The leverage ratio is \(\lambda = 2\).

(a) The general strategy is to go through the following steps:

- Set \(V_{t_0} = N_{t_0} = 100\). Calculate the floor

\[
F_{t_0} = \frac{V_{t_0}}{(1 + r_t)^5} = \frac{100}{(1 + 0.05)^5} = 78.35.
\]

- The cushion is:

\[
C_{t_0} = 100 - 78.35 = 21.65.
\]
- Amount to be invested in risky asset is \[ R_{t_0} = \lambda \times Cu_{t_0} = 2.65 = 43.30. \]

The iTraxx equity tranche which pays upfront cash of: \[ q_{t_0} \times N^{Eq} \]
where the Notional amount invested in the equity tranche is:
\[ N^{Eq} = \frac{R_{t_0}}{1 - q_{t_0}} = \frac{43.30}{1 - 0.14} = 50.35 \]

Hence we sell protection on Equity tranche with notional 50.35. The balance \( D_{t_0} = 100 - 43.30 = 56.70 \) is kept in a default-free deposit account enjoying Libor.

- As \( q_t \) changes over time \( t_1, t_2, \ldots \), the positions on risky assets and default-free bond investments are changed so as to keep the leverage ratio constant.

(b) Period 1: Calculate \( V_{t_1} \) as
\[
V_{t_1} = 100 \times (1 + L_{t_0}) + N_{t_0} \times (q_{t_0} - q_{t_1}) \\
+ q_{t_0} \times N_{t_0}(1 + L_{t_0}) + N_{t_0} \times 0.05 \\
= 105 + 50.35 \times (-0.01) + 7.05 \times (1 + 0.05) + 50.35 \times 0.05 \\
= 114.37.
\]

Hence
\[ F_{t_1} = \frac{100}{1.05^2} = 82.27. \]

Thus, the new
\[ R_{t_1} = 2 \times (V_{t_1} - F_{t_1}) = 2 \times 32.10 = 64.20. \]

With \( q_{t_1} = 15 \), the notional is \( N_{t_1} = \frac{64.20}{1 - 0.15} = 75.57 \)

Period 2: Calculate
\[
V_{t_2} = (V_{t_1} - \text{capital gains}) \times (1 + L_{t_0}) + N_{t_1} \times (q_{t_1} - q_{t_2}) \\
+ q_{t_1} \times N_{t_1}(1 + L_{t_0}) + N_{t_1} \times 0.05 \\
= 114.87 \times (1.05) + 75.57 \times (.15 - .05) + 0.05 \times 75.57 \\
\times (1 + 0.05) + 75.57 \times 0.05 \\
= 140.68.
\]
Hence

\[ F_{t_2} = \frac{100}{1.05^3} = 86.38. \]

Thus, the new

\[ R_{t_2} = 2 \times (V_{t_2} - F_{t_2}) = 2 \times 54.30 = 108.60. \]

With \( q_{t_2} = 5 \), the notional is

\[ N_{t_2} = \frac{108.6}{1-0.05} = 114.32. \]

**Period 3:** Calculate

\[ V_{t_3} = (V_{t_2} - \text{capital gains}) \times (1 + L_{t_0}) + N_{t_2} \times (q_{t_2} - q_{t_3}) \]
\[ \quad + q_{t_2} \times N_{t_2} \times (1 + L_{t_0}) + N_{t_2} \times 0.05 \]
\[ = 133.13 \times (1.05) + 114.32 \times (.5 - .16) + 0.16 \times 114.32 \]
\[ \quad \times (1 + 0.05) + 114.22 \times 0.05 \]
\[ = 152.13. \]

Here

\[ F_{t_2} = \frac{100}{1.05} = 95.24. \]

Thus, the new

\[ R_{t_2} = 2 \times (V_{t_2} - F_{t_2}) = 2 \times 56.89 = 113.78. \]

(c) *Cannot be calculated in this question. But a similar question has been answered in Ch20.*
Chapter 20

Principle Protection Techniques

1. The 1-year CDS rates in terms of basis points for the three IG names in the portfolio are given by:

\[ c(1) = 116, c(2) = 193, c(3) = 140. \]

The recovery rate is uniformly 40\%, i.e., \( R = 0.4 \). In every tranche a notional amount of $1.50 is invested.

(a) The default probabilities are given by:

\[
\begin{align*}
p_1 &= \frac{c(1)/10000}{1 - R} = \frac{0.0116}{1 - 0.4} = 0.0193333, \\
p_2 &= \frac{c(2)/10000}{1 - R} = \frac{0.0193}{1 - 0.4} = 0.0321667, \\
p_3 &= \frac{c(3)/10000}{1 - R} = \frac{0.0140}{1 - 0.4} = 0.0233333.
\end{align*}
\]

(b) To obtain the default distribution we need to get the default correlations between the three names first. If the correlations are all zero or all one (not possible in this case as the default probabilities are different) we can calculate the default distribution using standard probabilistic techniques. If they are different all zero or all one, then we can use the latent variable technique described in this chapter to generate dependent zero-one-valued random variables and simulate the default distribution from that.

(c) The defaults are uncorrelated. If \( D \) denoted the number of defaults,

\[
\begin{align*}
P(D = 0) &= (1 - p_1)(1 - p_2)(1 - p_3) = 0.9269757, \\
P(D = 1) &= p_1(1 - p_2)(1 - p_3) + (1 - p_1)p_2(1 - p_3) \\
&\quad + (1 - p_1)(1 - p_2) * p_3 = 0.07122975, \\
P(D = 2) &= p_1p_2(1 - p_3) + (1 - p_1)p_2p_3 + p_1(1 - p_2) * p_3 \\
&= 0.00178002, \\
P(D = 3) &= p_1 * p_2 * p_3 = 0.00001451.
\end{align*}
\]
(d) A 0 − 66% tranche provides protection on the first two defaults. The expected loss (in dollars) on this tranche is given by:

\[
L_{0-66} = 0 \times P(D = 0) + \left( \frac{1}{2} \times P(D = 1) + 1 \times P(D \geq 2) \right) \times 1.50 \\
\times (1 - R) = 0.03366846.
\]

(e) The pay over (in dollars) for the 0 − 50% and 50 − 100% tranches are:

\[
L_{0-50} = 0 \times P(D = 0) + 1.50 \times P(D \geq 1) \times (1 - R) = 0.06572185,
\]

\[
L_{50-100} = 0 \times [P(D \leq 1)] + \left( \frac{2}{3} \times P(D = 2) + 1 \times P(D = 3) \right) \\
\times 1.50 \times (1 - R) = 0.001081071.
\]

(f) We have \( c(1) = c(2) = c(3) = 100 \). Hence \( p = p_1 = p_2 = p_3 = \frac{100}{10000} = 0.01667 \). The default distribution is given by:

\[
P(D = 0) = (1 - p) = 0.98333,
\]

\[
P(D = 3) = 0.01667.
\]

There is only one tranche in this case which has spread: \( P(D = 3) \times 0.6 = 0.01 \), i.e., 100bp, whose expected loss is \$1.50 \times P(D = 3) \times 0.6 = $0.015.

2. Let the principal amount to be invested (in dollars) be \( N = 100 \). The iTraxx XO follows the path \{330, 360, 320\}. The Libor rate is fixed at \( L_t = 5\% \). The leverage ratio is \( \lambda = 2 \).

(a) The general strategy is to go through the following steps:

- Set \( V_{t_0} = N_{t_0} = 100 \). Calculate the floor

\[
F_{t_0} = \frac{V_{t_0}}{(1 + r_t)^5} = \frac{100}{(1 + 0.05)^5} = 78.35.
\]

- The cushion is:

\[
C_{u_{t_0}} = 100 - 78.35 = 21.65.
\]
• Amount to be invested in risky asset is

\[ R_{t_0} = \lambda \times C \times t_{t_0} \times 2.65 = 43.30 \]

which is the amount invested on iTraxx XO index. At the end of the year the spread received on the notional is \( 3.3 \times R_{t_0} = 1.43 \). The balance \( D_{t_0} = 100 - 43.30 = 56.70 \) is kept in a default-free deposit account enjoying Libor. At the end of the year the spread received on the notional is \( 3.3 \times R_{t_0} = 1.43 \).

• As the crossover index changes over time \( t_1, t_2, \ldots \), the positions on iTraxx XO and default-free bond investments are changed so as to keep the leverage ratio constant.

(b) Period 1

Calculate \( V_{t_1} \) as

\[
V_{t_1} = 100 \times (1 + L_{t_0}) + R_{t_0} \times c_{t_0})
\]

\[
= 105 + 43.30 \times 0.033)
\]

\[
= 106.43.
\]

Now

\[
F_{t_1} = \frac{100}{1.05^4} = 82.27,
\]

Thus, the new

\[
R_{t_1} = 2 \times (V_{t_1} - F_{t_1}) = 2 \times 24.16 = 48.32.
\]

Period 2: Calculate

\[
V_{t_2} = V_{t_2} \times (1 + L_{t_0}) + R_{t_1} \times c_{t_1})
\]

\[
= 117.75 + 48.32 \times (0.036)
\]

\[
= 113.49.
\]

Hence

\[
F_{t_2} = \frac{100}{1.05^3} = 86.38.
\]
Thus, the new
\[ R_{t_2} = 2 \times (V_{t_2} - F_{t_2}) = 2 \times 27.11 = 54.22. \]

**Period 3:** Calculate

\[ V_{t_3} = V_{t_2} \times (1 + L_{t_0}) + R_{t_2} \times (c_{t_2}) \]
\[ = 119.16 + 54.22 \times 0.032 = 120.90. \]

Here
\[ F_{t_3} = \frac{100}{1.05^2} = 90.70. \]

Thus, the new
\[ R_{t_3} = 2 \times (V_{t_3} - F_{t_3}) = 2 \times 30.200 = 60.40. \]

(c) Now there is a default and the index is \( c_{t_3} = 370. \) The notional investment iTraxx XO was 53.30 which becomes \( 53.40 \times \frac{29}{30} = . \)

An amount of \( \frac{60.40}{30} \) has to be paid, but with recovery rate 40%, \( \frac{60.40}{30} \times 0.40 \) is recovered. Hence

\[ V_{t_4} = V_{t_3} \times (1 + L_{t_0}) - \frac{R_{t_2}}{30} \times (1 - 0.4) + \frac{R_{t_2}}{30} \times \frac{29}{30} \times c_{t_3} \]
\[ = 120.90 \times (1.05) - \frac{60.40}{30} \times (1 - 0.4) + 60.40 \times \frac{29}{30} \times 0.037 \]
\[ = 127.90. \]
\[ F_{t_4} = \frac{100}{1.05} = 95.24. \]

Thus, the new
\[ R_{t_4} = 2 \times (V_{t_4} - F_{t_4}) = 2 \times 32.68 = 65.36. \]

3. The 1-year CDS rates in terms of basis points for the four IG names in the portfolio are given by:

\[ c(1) = 14, c(2) = 7, c(3) = 895, c(4) = 33. \]

The recovery rate is uniformly 30%, i.e., \( R = 0.3. \) In every tranche a notional amount of $1.00 is invested.
(a) The default probabilities are given by:

\[ p_1 = \frac{c(1)/10000}{1 - R} = \frac{0.0014}{1 - 0.3} = 0.002, \]
\[ p_2 = \frac{c(2)/10000}{1 - R} = \frac{0.0007}{1 - 0.3} = 0.001, \]
\[ p_3 = \frac{c(3)/10000}{1 - R} = \frac{0.0895}{1 - 0.3} = 0.1278571 \]
\[ p_4 = \frac{c(4)/10000}{1 - R} = \frac{0.0033}{1 - 0.3} = 0.004714286. \]

(b) The defaults are uncorrelated. Let \( D \) denote the number of defaults. Then

\[ P(D = 0) = (1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4) = 0.865429, \]
\[ P(D = 1) = p_1(1 - p_2)(1 - p_3)(1 - p_4) + (1 - p_1)p_2(1 - p_3)(1 - p_4) \]
\[ + (1 - p_1)(1 - p_2)p_3(1 - p_4) \]
\[ + (1 - p_1)(1 - p_2)(1 - p_3)p_4 = 0.1335727, \]
\[ P(D = 2) = p_1p_2(1 - p_3)(1 - p_4) + \cdots + (1 - p_1)(1 - p_2)p_3p_4 \]
\[ = 0.00099626 \]
\[ P(D = 3) = p_1p_2p_3(1 - p_4) + p_1p_2p_4(1 - p_3) \]
\[ + p_1p_3p_4(1 - p_2) + p_2p_3p_1(1 - p_1) = 0.000002069, \]
\[ P(D = 4) = p_1p_2p_3p_4 = 0.00000001205. \]

(c) The payment over Libor in an year (in dollars) for the 0–50%, 50–75% and 75–100% tranches are:

\[ L_{0-50} = 0 \times P(D = 0) + \left( \frac{1}{2} \times P(D = 1) + 1 \times P(D \geq 2) \right) \times (1 - R) = $0.04744927, \]
\[ L_{50-75} = 0 \times [P(D \leq 2)] + 1 \times P(D \geq 3) \times (1 - R) \]
\[ = $0.00000011449, \]
\[ L_{75-100} = 0 \times [P(D \leq 3)] + 1 \times P(D = 4) \times (1 - R) \]
\[ = $0.00000000084. \]
(d) We have \( c(1) = c(2) = c(3) = c(4) = 60 \). Hence \( p = p_1 = p_2 = p_3 = p_4 = \frac{60/10000}{1-0.3} = 0.008571429 \). The default distribution is given by:

\[
\begin{align*}
P(D = 0) &= (1 - p) = 0.9914286, \\
P(D = 3) &= 0.008571429.
\end{align*}
\]

There is only one tranche in this case which has expected pay over a year given by: \( $\left(P(D = 3) \times (1 - R)\right) = $0.006 \).

4. (a) If public debt leads to defaults in any of the items in the basket of protection sold, the investor has to pay up on that. But on the other hand the investor has hedged by buying protection on iTraxx crossover, which will guard against most of these defaults. Note that iTraxx crossover index is made up of 45 sub-investment grade (high risk of default) names. And they are most likely to default first. So it is more likely that the investor would receive payment from buying iTraxx crossover at 38bp than it would pay up for selling the CDS protection basket.

(b) If the OECD basket’s notional, say \( N \) increases by 20%, then the receipt from selling the CDS amounts to \( R_1 = N \times (1 + \frac{20}{100}) \times 0.0034 = 0.0041 \times N \). On the other hand payment by buying the iTraxx protection is \( P_1 = 0.0038 \times N \). Hence there is a positive carry in this trade amounting \( R_1 - P_1 = 0.0003 \times N \). A spread neutral position can be achieved by increasing \( N \) in OECD’s basket even less (12% would do).

(c) Since iTraxx is 0.65 correlated with OECD’s basket, default in one basket would be highly correlated with the default in the other. iTraxx crossover protects against the high risk names, and hence a default in the OECD’s basket is highly likely to occur with a default in the names in iTraxx crossover. Hence iTraxx acts as a good hedge for the basket.

5. done in Ch18.

6. The 1– year CDS rates in terms of basis points for the three IG names in the portfolio are given by:

\( c(1) = 56, c(2) = 80, c(3) = 137 \).
The recovery rate is uniformly 25%, i.e., \( R = 0.25 \). In every tranche a notional amount of $1.50 is invested.

(a) The default probabilities are given by:

\[
\begin{align*}
p_1 &= \frac{c(1)/10000}{1 - R} = \frac{0.0056}{1 - 0.25} = 0.0074667, \\
p_2 &= \frac{c(2)/10000}{1 - R} = \frac{0.0080}{1 - 0.25} = 0.0106667, \\
p_3 &= \frac{c(3)/10000}{1 - R} = \frac{0.0137}{1 - 0.25} = 0.0182667.
\end{align*}
\]

(b) To obtain the default distribution we need to get the default correlations between the three names first. If the correlations are all zero or all one (not possible in this case as the default probabilities are different) we can calculate the default distribution using standard probabilistic techniques. If they are different all zero or all one, then we can use the latent variable technique described in this chapter to generate dependent zero-one-valued random variables and simulate the default distribution from that.

(c) The defaults are uncorrelated. If \( D \) denoted the number of defaults,

\[
\begin{align*}
P(D = 0) &= (1 - p_1)(1 - p_2)(1 - p_3) = 0.9640094, \\
P(D = 1) &= p_1(1 - p_2)(1 - p_3) + (1 - p_1)p_2(1 - p_3) \\
&\quad + (1 - p_1)(1 - p_2)p_3 = 0.0355826, \\
P(D = 2) &= p_1p_2(1 - p_3) + (1 - p_1)p_2p_3 + p_1(1 - p_2)p_3 \\
&= 0.004065155, \\
P(D = 3) &= p_1p_2p_3 = 0.000001455.
\end{align*}
\]

(d) A 0 – 33% tranche provides protection on the first two defaults. The expected loss (in dollars) on this tranche is given by:

\[
\begin{align*}
L_{0-33} &= 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \\
&= 0.02699293.
\end{align*}
\]
(e) The payment over (in dollars) for the 0 – 33%, 33 – 66% and 66 – 100% tranches are:

\[
L_{0-33} = 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \\
= 0.02699293.
\]

\[
L_{33-66} = 0 \times (P(D \leq 1) + (1 \times P(D \geq 2)) \times (1 - R) \\
= 0.0003059777.
\]

\[
L_{66-100} = 0 \times [P(D \leq 2)] + (1 \times P(D = 3)) \times (1 - R) \\
= 0.0000109113.
\]

(f) Suppose the default correlations go up to \(\rho = 0.50\). We can use the latent variable model to get the distribution of \(D\) now. From our simulation we get

\[
P(D = 0) = 0.965273, \\
P(D = 1) = 0.033160, \\
P(D = 2) = .001502, \\
P(D = 3) = p_1 p_2 p_3 = 0.000065.
\]

(g) A 0 – 33% tranche provides protection on the first two defaults. The expected loss (in dollars) on this tranche is given by:

\[
L_{0-33} = 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \\
= 0.02604525.
\]

(h) The payment over (in dollars) for the 0 – 33%, 33 – 66% and 66 – 100% tranches are:

\[
L_{0-33} = 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \\
= 0.02604525.
\]

\[
L_{33-66} = 0 \times (P(D \leq 1) + (1 \times P(D \geq 2)) \times (1 - R) \\
= 0.00117525.
\]

\[
L_{66-100} = 0 \times [P(D \leq 2)] + (1 \times P(D = 3)) \times (1 - R) \\
= 0.0004875.
\]
8. The 1–year CDS rates in terms of basis points for the four IG names in the portfolio are given by:

\[ c(1) = 56, \quad c(2) = 80, \quad c(3) = 137, \quad c(4) = 12. \]

The recovery rate is uniformly 30%, i.e., \( R = 0.3 \). In every tranche a notional amount of $100.00 is invested.

(a) The default probabilities are given by:
\[
\begin{align*}
p_1 &= \frac{c(1)/10000}{1-R} = \frac{0.0056}{1-0.25} = 0.0074667, \\
p_2 &= \frac{c(2)/10000}{1-R} = \frac{0.0080}{1-0.25} = 0.0106667, \\
p_3 &= \frac{c(3)/10000}{1-R} = \frac{0.0137}{1-0.25} = 0.0182667, \\
p_4 &= \frac{c(4)/10000}{1-R} = \frac{0.0012}{1-0.25} = .0016
\end{align*}
\]

(b) To obtain the default distribution we need to get the default correlations between the three names first. If the correlations are all zero or all one (not possible in this case as the default probabilities are different) we can calculate the default distribution using standard probabilistic techniques. If they are different (not all zero or all one), then we can use the latent variable technique described in this chapter to generate dependent zero-one-valued random variables and simulate the default distribution from that.

(c) The defaults are uncorrelated. Let \( D \) denote the number of defaults. Then
\[
\begin{align*}
P(D = 0) &= (1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4) = 0.962467, \\
P(D = 1) &= p_1(1 - p_2)(1 - p_3)(1 - p_4) + (1 - p_1)p_2(1 - p_3) \\
&\quad \times (1 - p_4) + (1 - p_1)(1 - p_2)p_3(1 - p_4) \\
&\quad + (1 - p_1)(1 - p_2)(1 - p_3)p_4 = 0.03706809, \\
P(D = 2) &= p_1p_2(1 - p_3)(1 - p_4) + \ldots + (1 - p_1)(1 - p_2)p_3p_4 \\
&= 0.004616627 \\
P(D = 3) &= p_1p_2p_3(1 - p_4) + p_1p_2p_4(1 - p_3) \\
&\quad + p_1p_3p_4(1 - p_2) + p_2p_3p_1(1 - p_1) = 0.00000032375, \\
P(D = 4) &= p_1p_2p_3p_4 = 0.000000002328.
\end{align*}
\]
\[
L_{0-33} = 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \times 100 \\
= \$2.814974.
\]

(d) The payment over (in dollars) for the 0 – 33%, 33 – 66% and 66 – 100% tranches are:

\[
L_{0-33} = 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \times 100 \\
= \$2.814974.
\]
\[
L_{33-66} = 0 \times (P(D \leq 1) + (1 \times P(D \geq 2)) \times (1 - R) \times 100 \\
= \$0.03486769
\]
\[
L_{66-100} = 0 \times [P(D \leq 2)] + \left(\frac{3}{4} \times P(D = 3) + P(D = 4)\right) \\
\times (1 - R) \times 100 = \$0.0001822833.
\]

(e) The mezzanine tranche here is the one which protects against 33 – 66% of defaults. The value of this tranche (in bps) is:

\[
e^m = 0 \times (P(D \leq 1) + (1 \times P(D \geq 2)) \times (1 - R)
\]

To calculate the delta of this tranche we calculate \(e_m\) as above for the current \(iTraxxt(t)\) with the current values of \(p_1, p_2, p_3, p_4\). Then we increase \(iTraxxt(t)\) by 1% by increasing \(p_1, p_2, p_3, p_4\) by 1% and recalculate the default distribution of \(D\). Use this distribution to calculate new value of mezzanine tranche \(e^{m_{new}}\). Now the Delta of this tranche is the quotient of \((e^{m_{new}} - e^m)\) and \(\Delta iTraxxt(t)\).

(f) Suppose the default correlations go up to \(\rho = 0.50\). We can use the latent variable model to get the distribution of \(D\) now. From our simulation we get

\[
P(D = 0) = 0.965114, \\
P(D = 1) = 0.033412, \\
P(D = 2) = 0.001428 \\
P(D = 3) = 0.000046, \\
P(D = 4) = 0
\]
\[ L_{0-33} = 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \times 100 \]
\[ = $2.61645. \]

The payment over (in dollars) for the 0 − 33%, 33 − 66% and 66 − 100% tranches are:

\[ L_{0-33} = 0 \times P(D = 0) + (1 \times P(D \geq 1)) \times (1 - R) \times 100 \]
\[ = $2.61645. \]

\[ L_{33-66} = 0 \times (P(D \leq 1) + (1 \times P(D \geq 2)) \times (1 - R) \times 100 \]
\[ = $0.11055. \]

\[ L_{66-100} = 0 \times [P(D \leq 2)] + \left(\frac{3}{4} \times P(D = 3) + P(D = 4)\right) \times (1 - R) \times 100 = $0.0025875. \]

The mezzanine tranche here is the one which protects against 33 − 66% of defaults. The value of this tranche (in bps) is:

\[ c^m = 0 \times (P(D \leq 1) + (1 \times P(D \geq 2)) \times (1 - R) \]

To calculate the delta of this tranche we calculate \( c_m \) as above for the current \( iTraxx(t) \) with the current values of \( p_1, p_2, p_3, p_4 \). Then we increase \( iTraxx(t) \) by 1% by increasing \( p_1, p_2, p_3, p_4 \) by 1% and recalculate the default distribution of \( D \). Use this distribution to calculate new value of mezzanine tranche \( c_{m_{new}} \). Now the Delta of this tranche is the quotient of \( (c_{m_{new}} - c^m) \) and \( \Delta iTraxx(t) \).

9. (a) A Landesbank in Germany is an independent commercial bank which is backed by the regional government and acts as the central bank for a group of regional banks. Besides this, sometimes they also offer many products and services alike modern commercial banks.

For example, Landesbanks Baden-Württemberg (LBBW) is the central bank of the savings banks in Baden-Württemberg, Rhineland Palatinate and Saxony.
There are seven Landesbanks among the top 20 German banks with \textit{LBBW} in the fifth place.

Backed by the regional governments the Landesbanks tend to have high ratings for both long term and short term credit. For example, according to the S & P ratings in 2008: Landesbanks Baden-Württemberg : AA+/A-1+


(b) The LBBW is grandfathered and hence is guaranteed to provide the said returns on the holding. So if a trade neutral position is held by being short in DG Hyp 4.25% and long in LBBW 3.5% (under swaps), as spreads tighten, the gains from LBBW will remain the same but the payments for being short in DG Hyp will decline leading to eventual gains from holding this position.

(c) The same position can be replicated using CDS and a default-free deposit very simply. From the DG Hyp swap the investor pays coupons \( C_{t_1} = 4.25\% = c_{t_1} + s_{t_1} \text{(say)} \). On the other hand she receives \( Libor + 6.5\text{bp} \). Here 6.5 bp is the swap-rate spread \( s_{t_1} \). So on a nominal \( N \) the payment is \( N(c_{t_1} - Libor) \).

On the other hand LBBW swap results in receiving \( C_{t_0} = 3.5\% = c_{t_0} + s_{t_0} \text{(say)} \). The payment is \( Libor + 4.2\text{bp} \). Here 4.2 bp is the swap-rate spread \( s_{t_0} \). So on a nominal \( M \) the receipt is \( M(c_{t_0} - Libor) \). Hence total gain is

\[
G = M(c_{t_0} - Libor) - N(c_{t_1} - Libor) = Mc_{t_0} - Nc_{t_1} + (N - M)Libor.
\]

This position can be replicated by selling protection on LBBW 3.5% (which is secure by state guarantee) at a CDS rate \( c_{t_0} \) for a notional amount \( M \) and buying protection on DG Hyp 4.25% at CDS rate \( c_{t_1} \) on a notional amount \( N \). An amount \( M - N \) is deposited in a default-free account enjoying Libor.

Sell protection on LBBW 3.5%, and get into an IRS receiving Libor + spread, which is the swap rate, \( s_{t_0} \). Also

\( \text{(http://www.germanbanks.org/html/10_news/news_00.asp)} \)

10. A \textit{barbell} is a strategy of maintaining a portfolio of securities concentrated at two extremes in terms of maturity date: “very” short-term and “very” long term.
The investor has sold 7–year protection on the equity tranche and bought a barbell of 5-year and 10–year protection on the equity tranche. Since equity correlation has tightened for 7–year, the spread has widened. Look at the following graph showing spread vs. time for the equity tranche.

![Graph of spread vs. time](image)

**Figure 8: Spread of equity tranche(0 – 3%) vs. time**

The convexity position comes from the fact that we are receiving from the high volatility in the 7-year protection and paying less from the barbell strategy. Refer to the figure.
Chapter 21

CAPS/FLOORS AND SWAPTIONS WITH AN APPLICATION TO MORTGAGES

EXERCISES

Question 1

(a) The position of an agency which sells a callable coupon bond. We assumed that coupon bond has a maturity of 3 years and is callable only at the second year.

(b) The market thinks that short term volatility decreases as FED’s plans become clearer. The long term volatilities may increase because mortgage players hedge as FED cuts interest rates.

(c) A straddle is a “long call and long put” at the same strike price (See ch10, figure 10.13).

A swaption long straddle is a long position in the swaptions which are described above. Its cash flow is shown below:

So the traders take a long straddle position on the long dated swaptions and short straddle position on the short dated swaptions.

(d) If expectations are realized, both positions would be in the money. Trader has assumed a short position on short term volatility and short term volatility is lower. When this position is closed it would be in the money. The trader also has assumed a long position on the long term volatility and the long term volatility is higher so this position is in the money as well. So the positions can simply be unwound.

(e) The investor can make money only if long term volatility increases in the case of CSFB. Where as in the case of Lehman, even if long term volatility does not increase or even it decreases, earnings from short term volatility position would offset the losses.
Case Study: Danish Mortgage Bonds

(a) The following is a discussion based upon IFR, Special Report in issue 1239 during the Year 1998.

Danish mortgage bonds have long been domestic investors’ referred debt instrument. The combination of a high degree of security and a spread over government bonds means that Danish mortgage bonds form the foundation of the majority of domestic portfolios, professional as well as private investors.

In recent years, international investors have taken increasing interest in the Danish mortgage market and foreign holdings have expanded from DKK40bn (4%) in 1994 to approximately DKK75bn (7%) in the first quarter of 1998.

As a result of more widespread investor interest, most banks have had their mortgage bonds rated by one of the major US credit rating agencies. Moody’s Investors Service has assigned ratings of between Aa2 and Aa3 to mortgage bonds issued by Danish mortgage lenders. The fact that no Danish mortgage bank has ever defaulted on its obligations during their 200-year history substantiates the bonds’ high degree of safety.

(b) There are two major explanations for the increased international interest in Danish mortgage bonds.

First, international investors have become more technically oriented; most are no longer deterred from including complex, structured securities in their portfolios. The prospect of European Monetary Union has intensified this development in Europe. Many American investors in particular find that the Danish sector for mortgage bonds is very similar to their domestic mortgage-backed market. Second, the Danish mortgage bond market offers a number of attractive investment openings that may increase the return on a diversified portfolio of debt securities.

With approximately DKK1,013bn in circulation, the market is relatively large, even by international standard. It is very much the market for government bonds, which has attracted foreign investors for many years and has an outstanding volume of DKK670bn. A number of
classes of mortgage bonds can compare with the biggest series in the world.

The table below outlines the major features regarding the most liquid Danish mortgage bonds, although there are transactions with other features:

**Coupon** 4%-8%

**Life** 1, 10, 20 or 30 years

**Annual repayments** 4 and 1

**Repayment profile:** Annuity payments but also bullet bonds and serial bonds. Most bonds may be prepaid at par. Termination of a loan generally requires a two-month notice prior to the date of payment.

**Market making** For the most liquid bonds. Some 15 brokers are under an obligation to quote amounts of typically DKr25m to DKr50m.

**Futures** Futures exist in a basket of the 6% bond of 2029 and the 7% of 2029. The Danish market for futures and options is relatively illiquid.

(c) Pricing of Danish mortgage bonds is dominated by the prepayment element. Assessment of future redemption rates is the predominant factor in connection with analysis and pricing. As a result, Danish stockbrokers have a long tradition of analysing and assessing callable bonds and have developed standard pricing methodology. In this context the mortgage banks, via the Copenhagen Stock Exchange, provide a number of data, such as an outline of debtor distribution and weekly prepayments to support the assessment of the prepayment volume.

International investors tend to invest in bonds with the lowest prepayment risk. Most prefer low-coupon bonds, which also reduce the probability of a negative convexity.

The expected additional yield compared with government bonds depends among other things on coupons, lifetime and liquidity. The spread to government bonds is relatively high, which means the option adjusted spread (OAS) on a number of benchmark bonds is approximately 70bp compared with an additional yield of only some 20bp to 25bp on the longest German jumbo Pfandbriefe.
(d) Pricing of Danish callable bonds is, generally speaking, based on assumptions about the likelihood that the debtor will call the bonds under different interest rate scenarios. A binomial model describing the future direction of interest rates combined with distribution assumptions describing debtors’ behaviour patterns in the given interest situation is generally applied. The behaviour relating to calls is normally based on the debtors’ profit claims; this is typically assumed to be log-normally distributed. A number of key ratios such as theoretical price, option-adjusted duration, option-adjusted spread and more can be calculated on the basis of the defined binomial tree and the calculated prepayment gains.

In addition to advanced option models, CPR models and various equilibrium models are also applied to some extent.

(e) The Danish bond market is characterised by precisely the same trade conventions and settlement procedures for government and mortgage bonds. Consequently, most domestic brokers are able to trade government and mortgage bonds at the same desk. Mortgage bonds in Ecu/euro As of June 2 1998, Nykredit has issued bonds denominated in Ecu that will automatically be redenominated in euros as of January 1 1999.

The range of loans includes non-callable bullet loans with maturities of 1 to 11 years with coupons of 4% and callable annuity loans with maturities of 10, 20 and 30 years carrying coupons of 4% for bonds due between 2008 and 2018, 5% until 2028 and 6% for even longer securities.
Chapter 22
ENGINEERING OF EQUITY INSTRUMENTS
EXERCISES

Question 1

(a) Let’s assume that the firm may default only on last coupon payment
date and that when this happens stock price would be less than some
predetermined price $K$ at the expiration date.

Cash flow of reverse convertibles would then look like as follows.
(b) We need a short position on the put option.

The yields on reverse convertibles are higher since their price are less than a coupon bond with the same coupon payments.

(c) Firm chooses to exercise the put option if stock price is less than say, $K$. So, bond holders receive the stock when stocks have in fact fallen, under adverse conditions. They probably will prefer to cash in their shares which may further push down the stock prices.

(d) There is no principal protection in risky bonds. But this is different. Unless the company defaults, independent from the company’s performance, investors would get their principal. In the case of reverse convertibles, however, even if company does not default, investor may suffer from substantial losses.
Chapter 22

CASE STUDY: VOLATILITY TRADING

(a) The readings in this case study deal with Convertible and Reverse-Convertible bonds. These are interesting instruments by themselves, but this Case Study begins with some volatility models, since these types of structured equity products are essentially instruments of volatility.

Let the “volatility” at time $t$ of a stock be denoted by $\sigma_t$ and let this be a random process. Then the mean reverting model for this stochastic process will be given by:

$$d\sigma_t = \lambda(\mu - \sigma_t)dt + \gamma\sigma_t dW_t$$

Here the $\lambda, \mu, \gamma$ are various parameters of this model. The $\gamma$ is the volatility of volatility. The $\mu$ is average long-run volatility and the $\lambda$ is the rate of convergence of the spot volatility towards this long-run average.

The first reading deals with an example of how this type of model can be used in position taking.

(b) These are discussed in the text on pages 501–503. The decomposition of a convertible bond is shown in Figure 17-3.

(c) Here is a recent example of a convertible bond, as mentioned in the IFR, November 2004.

Yet another Asian convertible bond (CB) was at the centre of controversy last week, as a US$125m CB for CMC Magnetics began life amid a haze of revised terms and a convoluted post-launch repricing.

The bought deal was launched by lead manager JP Morgan late in the afternoon of October 29, as a fixed-priced, zero-yield deal at a conversion premium 15% above the stock’s close that day. By the end of the evening, the deal had priced with a 0.8% yield, a 20% conversion premium and an October 28 reference price.

(...) After launching on the initial terms, JP Morgan realised that the conversion premium fell outside terms filed with the Securities and Futures Bureau (SFB), which had promised a 20%-50% premium.
The bank was thus obliged to return to investors with revised terms, lifting the conversion premium to 20% while compensating investors with the 0.8% yield to put/maturity.

The new deal also used a lower reference share price. The original term sheet used the closing price on October 29 (NT$14.9); the final one referenced to the close on October 28 (NT$14).

The conversion premium is the amount (either in percentage terms or in dollar amount), by which the conversion price of a convertible security exceeds the current market value of the underlying common stock.

If the bond is converted, then the issuer needs to issue new stocks. This will have a dilution effect on the existing positions.

(d) A convertible bond contains a call option. The investor has in a sense purchased an embedded call. If the price of the equity exceeds the conversion price then the investor will “call” the stocks.

In a reverse-convertible bond it is the issuer who has purchased an option. In fact this is a put option. The issuer determines if and when to convert the bond into stock. The investor on the other hand is short an embedded put. The investor will accept the delivery of a bond or a stock at a pre-determined price if the issuer chooses to “convert”.

For this added risk, the investor will receive a higher coupon.

(e) When volatility increases, it gives the following opportunities to dealers. High volatility implies high option prices. Thus reverse convertibles can be structured with higher coupons. This attracts investors.

At the same time, the issuing company will be long an option. By hedging this the company can isolate the gamma. Hence, if the option is purchased at a “reasonable” price from the investor (which is quite likely in such cases...) then the gamma gains can very well exceed the premium paid for the option.

The structurers gain two ways. From higher volatility and from selling new instruments.

(f) The Chapter shows how to construct synthetic convertibles and reverse-convertibles. The general principle is illustrated in Figure 17-3.
Regulators may worry that such instruments are making investors sell options. Many investors may not realize how to price options given a certain volatility structure. Under such conditions they may sell options below the fair price.